On Singular Quantification in Japanese

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Chapter 1

Indeterminate phrases, interrogative construction, and quantification

Japanese has two sorts of quantificational devices. One type of quantification is realized by quantity phrases like “subete” (all), “ta-suu” (a large number of) and “dai-bubun” (large part of, most). They appear either as part of a noun phrase like (1)–(3), or as an adverbial like (4).

(1) Subete no gakusei ga kita.  
    all GEN student(s) NOM came.  
    (All students came.)

(2) Gakusei no subete ga kita.  
    student(s) GEN all NOM came.  
    (All of the students came.)

(3) Gakusei subete ga kita.  
    student(s) all NOM came.  
    (All students came.)

(4) Gakusei ga subete kita.  
    student(s) NOM all came.  
    (The students came all.)

Another type of quantification in Japanese is realized by a combination of a word which corresponds to an English wh-word and one of the two particles “mo” and “ka”. Here are two simple examples.
It is generally recognized that “mo” works as a universal quantifier while “ka” works as a particular (existential) quantifier. They work as quantifiers, however, only with the presence of Japanese wh-words like “dare” as in the above examples\(^1\).

One striking feature of this type of quantification is that the words which express quantification together with quantifiers “mo” or “ka” make interrogative sentences, when they are used with the particle “ka” signalling interrogative mood\(^2\). That is the reason why these words are said to correspond to English wh-words.

Hence, Masuoka and Takubo ([Masuoka and Takubo 1993, p.39]) call such words “interrogatives”. Besides “dare”, they list the following as interrogatives\(^3\):

\(^1\) It is worth remarking that “mo” and “ka” are each used to form conjunction and disjunction respectively as in the following examples.

(i) Taro mo Hanako mo Jiro mo waratta.  
(Taro, Hanako and Jiro laughed.)

(ii) Taro ka Hanako ka Jiro (ka) ga waratta.  
(Taro or Hanako or Jiro laughed.)

For further details, see the Appendix to [Iida 2008].

\(^2\) We suppose that the interrogative “ka” is a different word from the existential “ka”.

\(^3\) “Dono” and “donna” are always used with some noun N. Roughly speaking, “dono N” means “which N”, and “donna N” means “what N”. However, this is a very rough characterization. For one thing, in many uses of “donna N” a certain sort of modality is involved. Consider the following pair of sentences.

(i) Taro wa dono gakusei mo oshieta.  
(Taro taught every student.)

(ii) Taro wa donna gakusei mo oshieta.  
(Taro taught any student.)
If the quantifier “mo” or “ka” is put after any of these expressions, it results in something like a sorted quantifier.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dare mo</td>
<td>everyone, anyone</td>
</tr>
<tr>
<td>Doko mo</td>
<td>everywhere, anywhere</td>
</tr>
<tr>
<td>Dono N mo</td>
<td>every N, any N</td>
</tr>
<tr>
<td>Itsu mo</td>
<td>always, any time</td>
</tr>
<tr>
<td>Dare ka</td>
<td>someone</td>
</tr>
<tr>
<td>Doko ka</td>
<td>somewhere</td>
</tr>
<tr>
<td>Dono N ka</td>
<td>some N</td>
</tr>
<tr>
<td>Itsu ka</td>
<td>sometime</td>
</tr>
</tbody>
</table>

Masuoka and Takubo have a special name for such expressions; they call them “fu-tei go (indeterminate words)” ([Ibid.]).

In this paper, however, I would like to adopt a different terminology. I am going to apply the term “indeterminate words” to what Masuoka and Takubo call “interrogatives” (Japanese wh-words)\(^4\), and “indeterminate phrase” for a phrase which contains an indeterminate word without counting the particles “mo” and “ka” as its part.

Indeterminate words belong to a larger category of words which comprise the words that are used in demonstrative expressions. These demonstrative words (“shiji-go” in Japanese grammar), which usually come in three varieties with characteristic sound “ko”, “so”, “a” and with indeterminate words form a series that is referred as “ko-so-a-do series” as the following table shows.

<table>
<thead>
<tr>
<th>Indeterminate Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>kore, sore, are, are</td>
</tr>
<tr>
<td>kono, sono, ano, ano</td>
</tr>
<tr>
<td>komma, sonna, anna, anna</td>
</tr>
<tr>
<td>koko, soko, asoko, asoko</td>
</tr>
</tbody>
</table>

I am going to be mostly concerned with noun phrases which contain indeterminate words, and I distinguish among them simpler indeterminate noun phrases and more complex indeterminate noun phrases. Some of indeterminate words like “dare” and “doko” are by themselves indeterminate noun phrases, and some like “dono” and “donna” become noun phrases only when they are put before a noun N. Those phrases of the form “dono + N” and “donna + N” with a noun N, together with indeterminate words like “dare” and “doko”, will be called “indeterminate terms”.

In (i), the extension of “gakusei” is certain actual students given in the context, while that of “gakusei” in (ii) need not be confined to actual students. If we want to convey the nuance of (ii), it is better to use a sentence with modal auxiliary like “Taro would teach any student”.

\(^4\) Shimoyama’s term for them is “indeterminate pronouns” ([Shimoyama 2006]). But I hesitate to adopt it, because words such as “dono” and “dou” are not pronouns.
An indeterminate word may occur in a noun modifying clause. A noun phrase which contains an indeterminate noun in its modifying clause is an indeterminate noun phrase\(^5\). The following table shows this classification of indeterminate noun phrases.

<table>
<thead>
<tr>
<th>Indeterminate noun phrases</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. indeterminate terms</td>
</tr>
<tr>
<td>dare</td>
</tr>
<tr>
<td>dono hon</td>
</tr>
<tr>
<td>donna hito</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>2. indeterminate noun phrases other than indeterminate terms</td>
</tr>
<tr>
<td>dare no hon</td>
</tr>
<tr>
<td>dono hon o yonda hito</td>
</tr>
<tr>
<td>itsu kita hito</td>
</tr>
<tr>
<td>...</td>
</tr>
</tbody>
</table>

Generally speaking, an indeterminate term has only one occurrence of a noun, while other more complex indeterminate noun phrases have more than one occurrence of a noun, which may be the same noun as in “dono oya no oya” (a parent/parents of which parent(s)).

Just as an indeterminate term “dare” appears both in a question (7) and quantified sentences (5) and (6), an indeterminate phrase “dare no hon” may appear in an interrogative sentence\(^6\) as well as a quantified sentence.

\[
\begin{align*}
(8) & \text{ Dare no hon ga omoshiroi- desu ka.} \\
 & \text{ who GEN book(s) NOM interesting POL ?} \\
& \text{(Whose book/books is/are interesting?)}
\end{align*}
\]

\[
\begin{align*}
(9) & \text{ Dare no hon mo omoshiroi- desu.} \\
 & \text{ who GEN book(s) } \forall \text{ interesting POL} \\
& \text{(Every author’s book is interesting.)}
\end{align*}
\]

An indeterminate phrase “dare no hon” indicates what is asked in (8), while the same phrase functions as a restrictor for a universal quantification in (9).

The reason why I adopt the present terminology is that I wish to emphasize the fact about Japanese that interrogative construction and quantification

\(^5\) This characterization is not accurate, however. A noun phrase might contain an indeterminate noun and yet is not an indeterminate noun phrase. This happens when all the indeterminate nouns contained in it are bound by “mo” or “ka”. For a more exact definition of indeterminate noun phrases, please see §4.2.

\(^6\) In general, you should use a polite word ending in asking a question in Japanese. Hence, (8) has a copula in polite form (POL). I put (9) also in polite form for a comparison.
utilize exactly the same expression. For that purpose, we need a term that designates the syntactic element common to interrogative construction and quantification, and “indeterminate phrase” seems to be such a term. Let me give another pair of examples that show that exactly the same expression is used for a question and a quantification.

(10) Itsu kita hito ga ayashii desu ka.
    when came person NOM suspect POL ?
    (Which person who came when is suspect?)

(11) Itsu kita hito mo ayashii desu.
    when came person ∀ suspect POL
    (Every person is suspect, no matter when she came)

In this paper, my main concern is with an indeterminate phrase beginning with “dono”. I consider how such a phrase gives rise to singular universal quantification when it is combined with the universal quantifier “mo”, which is not itself a singular quantifier.

Shimoyama ([Shimoyama 2006]) offers a convincing account of “dono” quantification, and I owe greatly to it. In particular, I owe two basic insights to Shimoyama’s work. First, the entire noun phrase preceding “mo” works as a restriction of universal quantification. Secondly, a unified account should be given for indeterminate phrase quantification and interrogative constructions with indeterminate phrase. I hope the present inquiry is a development of her seminal work.

Of course, there are a number of differences in detail between her account and mine. First, my account is set in the framework of plural logic, which will be described in the following chapter. Secondly, in that framework, I try to describe and explain how “dono” quantification interacts with other type of quantification that may occur in the same sentence. This needs an account of quantity phrase quantification and I supply it as much as necessary for the present purpose in chapter 3. Other differences will become clear as I proceed.
Chapter 2

Plural logic as a metalanguage for Japanese semantics

It is almost a commonplace that there are two distinctions among English noun phrases that are signalled by their form.

(A) denoting some definite objects (definite) / indicating indefinitely some objects within a certain domain (indefinite)
(B) denoting or indicating a single object (singular) / denoting or indicating a number of objects (plural)

Japanese is entirely different in this respect. It has neither definite nor indefinite articles, and there is no distinction among its noun phrases between singular and plural, either.

Let us take a very simple sentence of Japanese.

(12) Gakusei ga waratta.
student(s) NOM laughed.

If we are shown this sentence without any specification of a context, all of the following are its possible translations into English.

(12a) The student laughed.
(12b) A student laughed.
(12c) The students laughed.
(12d) Some students laughed.
Without a context, the noun phrase “gakusei” in (12) may be definite or indefinite, singular or plural.

There is a big difference between the distinction of definite/indefinite and that of singular/plural, however. In Japanese also, given a context, it is usually clear whether a noun phrase is definite or indefinite; if you cannot determine which, you will not understand what is said by a sentence that contains its occurrence. In contrast to this, it is frequently impossible to judge whether a given occurrence of a noun phrase is supposed to denote (indicate) a single thing or a number of things, even when the context is clear. And yet, in most cases, there will be no difficulty in understanding what is said by the sentence with that noun phrase.

This shows that there is no need for a Japanese noun phrase to indicate whether it refers to a single thing or a number of things. A Japanese noun phrase is a number-neutral expression. The same thing applies to a Japanese verb phrase and adjectival phrase. Japanese verbs and adjectives do not change their forms according to the number. Japanese is a language which is mostly number-neutral.

In the current practice of formal semantics, you frequently encounter a semantic account of some natural language expressions given in a sort of regimented natural language as a metalanguage. It also usually happens that this metalanguage is the standard language of modern logic, namely the first-order language of predicate calculus, together with set-theoretic apparatus and some non-logical vocabulary that is necessary to interpret the object language vocabulary.

It is noteworthy that this standard language of modern logic is not very good at handling various plurality phenomena in natural language. The standard language in logic is not a number-neutral language; in this language, reference and predication relations, which are the two basic semantical relations in any language, are both singular. Thus, a name only refers to single things, and never to a number of things, while a predicate is true of each of the things in its extension, and never true of a number of things, as it were, at the same time. Consequently, a variable is singular as well.

It is almost perverse to use this language, in which plurality is completely banished, in order to give a semantic account of a natural language with various plural constructions. However, if you wish to stick to the standard language and logic even here, there seems to be a way of doing so. It is to appeal to “plural objects”, which is a common practice found in the works by linguists today. There are several options here; plural objects might be sets, mereological sums, or elements of a certain algebra. Whichever you choose, those “plural objects” are each single objects that purport to represent a number of things. Hence, the appeal to “plural objects” should be characterized as turning plurality of things into a single object of special sort in order to apply the standard logic to
This way of proceeding is sometimes called “singularist strategy”. The standard language has also the same difficulty with a number-neutral language like Japanese. As we saw above, a Japanese noun “gakusei” may denote a number of students as well as a single student. Hence, singularist strategy has also been taken in semantical studies of Japanese. Thus, the extension of a Japanese noun “gakusei” is taken to consist of, say, any mereological sum of students as well as individual students.

Given the number-neutral character of Japanese, however, the most natural option is to stop using the standard language and adopt a more suitable language as its metalanguage; as a matter of fact, we have such a language in the language of plural logic, which has been developed by several authors. But it must be remarked that the name “plural logic” is misleading, because it is, in reality, a number-neutral logic; “plurality” here is understood to include singularity as a special case.

I believe that adopting plural logic as our metalanguage is the only feasible option that is currently available. For, we can argue that singularist strategy has fatal flaw.

It is not because singularist strategy involves the introduction of the “plural objects” into our ontology, while we do not need to recognize any new entities if we adopt plural logic. For, this may not constitute a good argument against singularist strategy. In many parts of our language, it becomes an issue whether we should change our logic or expand our ontology in order to give an account of relevant expressions. For example, it has been debated whether we should admit possible worlds as part of our ontology, or recognize our modal locutions as primitive and expand our logic into modal logic for a satisfactory account of our modal talk.

It seems that there is no general rule in deciding such issues. It does not seem to be true that changing our logic is always preferable to expanding our ontology, or vice versa. A case in point is Davidsonian analysis of action sentences, in which a new kind of entities, namely, events, are introduced into our ontology, instead of looking for some new logic for action sentences. Now Davidsonian analysis is widely accepted in both philosophy and linguistics. In this case, the general consensus seems to be that expanding our ontology is a better strategy than changing our logic.

We might argue, however, that the case of plurality is very different from that of action sentences. The most obvious complaint to singularist strategy is that it fails to take plurality at its face value; under this strategy, plurality is turned into singularity, which is its opposite. Davidsonian analysis does nothing like this to action sentences. Although some philosophers object to it, saying that it is a mistake to identify an action as a special kind of event, even they admit that there is a close connection between them.

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1. [Oliver and Smiley 2001].
2. See [Hossack 2000], [McKay 2006], [Oliver and Smiley 2001], [Yi 2005] and [Yi 2006].
3. This strategy is sometimes called “modalism”.

The main reason to resist singularist strategy is that it turns plurality into its opposite, and hence, that it inevitably results in either ascribing a wrong truth condition to a sentence which contains plurality in an essential way, or having to retain plurality in the metalanguage\textsuperscript{4}. Consider the sentence

\begin{equation}
\text{Taro and Hanako met together.}
\end{equation}

This sentence essentially contains plurality, because it cannot be paraphrased into a combination of singular sentences. For example, if we try to analyze it as a simple conjunction, then it results in a nonsense like this.

\begin{equation}
\text{\textasterm{Taro met together} \land \text{Hanako met together}.}
\end{equation}

According to singularist strategy, (13) is interpreted as a sentence in which a predicate “\textit{x met together}” is applied to a plural object denoted by “Taro and Hanako”. Then, the truth condition of (13) would be one of the following, depending on how plural objects are construed.

\begin{itemize}
  \item[(15a)] The set \{Taro, Hanako\} met together.
  \item[(15b)] The mereological sum consisting of Taro and Hanako met together.
  \item[(15c)] The group consisting of Taro and Hanako met together.
\end{itemize}

It is obvious that all of these give the wrong truth condition to (13), simply because they are all false while (13) can be true. (15a)–(15c) are doubly wrong. First, although groups might meet together, sets and mereological sums are not the things that can meet together, and secondly, it needs more than two things for meeting together to take place, while only a single thing is mentioned in (15a)–(15c). Two groups might meet together, but a single group cannot meet together.

This shows that the predicate “\textit{x met together}” should be reinterpreted if we wish to apply singularist strategy to (13). Instead of “\textit{x met together}”, we should have a new predicate “\textit{x met-together*}”, which can be true of a single set, mereological sum, or group. Then, the truth condition of (13) would be given, for example, by the following sentence.

\begin{equation}
\text{The set \{Taro, Hanako\} met-together*}. \end{equation}

An obvious question here is how this predicate that is newly introduced into our metalanguage is explained. It would run like this.

\begin{equation}
\text{\textit{a} satisfies the predicate “\textit{x met-together*}” if and only if the members of \textit{a} met together.}
\end{equation}

\textsuperscript{4} The following argument is an adaptation of the one found in [McKay 2006, p.24]. Also see a relating discussion in [Oliver and Smiley 2013, pp.37ff.].
This makes it clear that we need a plural description “the members of a” and a plural predication “met together” in our metalanguage to make (16) intelligible to us. Hence, without a prior understanding of plural locution, we cannot understand a truth condition assigned to a sentence reinterpreted according to singularist strategy. In other words, singularist strategy cannot succeed in analyzing plurality away.

We have conducted our argument with an example from English, but exactly the same argument can be given with a corresponding Japanese sentence

\[(18) \text{ Taro to Hanako ga atta.} \]
\[\quad \text{and NOM met}\]

(Taro and Hanako met together.)

Given such a limitation of singularist strategy, it seems strange that it is very popular among linguists. I suspect it is because many linguists working in semantics believe that it is enough to give a truth condition relative to a model\(^5\). If we were only concerned with giving a truth condition relative to a model, we might be satisfied with (16), provided that some element of the model is regarded as “the set \{Taro, Hanako\}” and some set in the model is assigned as an extension of the predicate “met-together\(^6\)”. However, giving a truth condition of a sentence relative to a model is not the same as giving its truth condition itself. If the object of model-theoretical study is a natural language like English or Japanese, there must be an intended model which is supposed to give the actual meanings of the expressions that belong to it. In particular, the actual meaning of a sentence is its truth condition relative to the intended model.

Therefore, if we wish to give a semantic account of a part of Japanese that contains (13), we should not be satisfied with (16); we should specify what truth condition is assigned to (13) in the intended model of the language. Then, we will realize that singularist strategy has done nothing to further our understanding of plurality.

We conclude that the language of plural logic is not only useful to do semantics for a number-neutral language like Japanese but also essential to avoid the temptations of singularist strategy.

Plural logic is an extension of standard first-order logic and it is not difficult to guess where such an extension is necessary and how it is done.

First, let me introduce some notations. We use

\[X, Y, Z, \ldots\]

as plural variables. Their values are a number of objects, including one, found in the domain. If we wish such a variable to take only single objects as its value,

\(^5\) For a relevant discussion, see [Iida 2002, §4.4].
we use the special sort of plural variables

\[ x, y, z, \ldots \]

They are called singular variables.

The language of (first-order) plural logic is different from that of the standard first-order logic only in two respects: (i) its variables are plural ones, and (ii) it has a two-place logical predicate “\( \eta \)” instead of the (singular) identity predicate “=”. 

\[ X \eta Y \]

means that \( X \) are among \( Y \).

Plural identity “\( X \equiv Y \)” is defined by the following.

\[ X \equiv Y \iff X \eta Y \land Y \eta X. \]

We also use the predicate “\( I X \)” which means that \( X \) is an individual. It can be defined by

\[ I X \iff \forall Y[Y \eta X \rightarrow X \eta Y]^6. \]

Instead of translating a Japanese sentence into a language of plural logic, we are going to give its truth condition in a metalanguage which is the language of plural logic. Only regimentation we perform on a Japanese sentence is to put parentheses in order to show its phrase structure. A theory of Japanese semantics consists of a number of semantic axioms formulated in the language of plural logic and their consequences derived in the same language.

Our style of giving semantics of an expression is by specifying the condition for the objects to be its semantic values. We have a predicate in our metalanguage which is written like

\[ \text{Val}(X, \alpha), \]

and read as “\( X \) are among the semantic values of an expression \( \alpha \)”. For example, semantics of a Japanese noun “gakusei” is given in the following statement.

\[ \text{Val} (X, \text{"gakusei"}) \iff X \text{ is a student / are students.} \]

If our metalanguage is Japanese, we get a more homophonic condition, because a Japanese noun “gakusei” is number-neutral and has the same form whether it is applied to a single student or a number of students.

\[ \text{Val} (X, \text{"gakusei"}) \iff X \text{ wa gakusei da.} \]

This clearly shows the merit of giving a semantic account of a number-neutral language in a number-neutral metalanguage.

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6 [McKay 2006, p.120].
Chapter 3

An outline of the semantics of quantity phrase quantification

Here I give a brief sketch of the semantics of quantity phrase quantification, which is one of the two major types of quantification in Japanese\(^1\); the other major type is indeterminate phrase quantification, to which the subject of this inquiry belongs. I hope this sketch shows how the adoption of plural logic helps us in doing Japanese semantics.

A quantity phrase is formed from a noun which belongs to a class of nouns which Masuoka and Takubo call “quantity nouns” ([Masuoka and Takubo 1993, pp.34f.]). A quantity noun can appear as part of a noun phrase, modify a verb phrase, or form a nominal predicate.

I classify quantity phrase quantification into two categories, one of which is further divided into three subcategories. Thus, the general picture is as follows.

(A) Quantity phrases that function as non-proportional quantifiers,

(A–1) which are monotone increasing,

(A–2) which are monotone decreasing, and

(A–3) which are neither (non-monotone).

(B) Quantity phrases that function as proportional quantifiers.

If I may state the conclusion in advance, both of these quantity phrases can be regarded as first-order plural predicates, that is, predicates that can be true

\(^1\) Some of the material in this chapter is found in [Iida 2007], although it was given in the singularist framework, to which I no longer adhere. Quantity phrase quantification is also a major means of expressing mass quantification in Japanese. In this sketch, I discuss only quantification over countable objects.
of a number of individuals; the difference between them is that those expressing non-proportional quantifiers are unary (one-place) predicates in contrast to those expressing proportional quantifiers that are binary (two-place) predicates.

3.1 Quantity phrases that function as non-proportional quantifiers

Here are three typical examples of this sort of quantification.

(19) Go-nin no gakusei ga waratta.
      five-CL GEN student NOM laughed
      (Five/The five students laughed.)

(20) Gakusei shou-suu ga waratta.
      student small number NOM laughed
      (A/The small number of students laughed.)

(21) Gakusei ga chyoudo san-nin waratta.
      student NOM exactly three CL laughed
      (Exactly three students laughed.)

These examples show that there are three patterns in this sort of quantification. If we consider only simple cases where “Q” stands for a quantity phrase, “N” for a noun, and “V” for a verb, then each of (19)–(21) exemplifies one of the three patterns. (There is also another pattern in which a quantity phrase appears in a nominal predicate. This pattern will be discussed later in §3.3.)

(I) Q + no + N + ga + V. (19)
(II) N + Q + ga + V. (20)
(III) N + ga + Q + V. (21)

In the first two patterns, a quantity phrase is part of a noun phrase, while it appears within a verb phrase as an adverbial in (III).

These three examples also exemplify the three kinds of non-proportional quantifiers that differ in their semantic properties.

(i) monotone increasing: “go-nin” (five persons–(19)), “ooku” (many), “ta-suu” (a large number), “suu-nin” (a few persons), “san-nin iyou” (more than two persons), etc.

(ii) monotone decreasing: “shou-suu” (a small number–(20)), “sukoshi” (few, little), “san-nin ika” (less than four persons), etc.

2 “CL” is an abbreviation for “classifier”.

15
(iii) non-monotone: “chudo san-nin” (exactly three persons–(21))

The two classifications are independent from each other. Thus, any phrase in (i)–(iii) can occur in any of the patterns (I)–(III). You can convince yourself of it, if you take each of the quantity phrase in (19)–(21) and make it a subject of the sentences of the remaining two other patterns.

3.1.1 Pattern (I) with an indefinite subject
Let us start with (19). As is indicated in its English translation, its subject “go-nin no gakusei” may either indefinite or definite depending on the context. In either case, the quantity phrase “go-nin” is construed as a predicate that is true of \( X \) that is at least five in number\(^3\). The truth condition of (19) with an indefinite subject can be given by the following sentence, which might be regarded as a sentence in regimented English with some notations from plural logic.

\[
\exists X [\text{Five (}X\text{)} \land \text{Students (}X\text{)} \land \text{Laughed (}X\text{)}]
\]

However, we should work with a Japanese sentence itself. First, we formally represent (19) in the following way.

\[
(\text{23}) \ ((\text{go-nin no (gakusei)}) \ \text{ga}) \ \text{waratta}
\]

A Japanese particle “no” has many functions. It functions here as a particle that connects a noun-like adjectival phrase with a common name in order to form a complex noun phrase, which has a pattern such as

\[
(\text{AdjectivalNoun no (CommonNoun)})^4.
\]

The semantics of such a phrase is given by

**Axiom of noun modification**

\[
\text{Val}(X, \ “(AN \ no \ (CN))” \) \ \Leftrightarrow \ \text{Val}(X, \ AN) \ \land \ \text{Val}(X, \ CN).
\]

---

\(^3\) I suppose that a classifier “-nin” does not contribute to the truth condition of a sentence in which it occurs. Hence, “go-nin” and “go-dai” as in “go-dai no kuruma” (five cars) are regarded as having the same semantic value. Of course, this needs an argument, which I do not offer here.

\(^4\) Another main function of “no” is to indicate an argument of a relational noun like

\[
\text{Hanako} \ \text{no} \ \text{oyna} \ \text{GEN} \ \text{parent(s)}
\]

(Hanako’s parent/parents)

In this example, an argument of a relational noun “oya” is filled by “Hanako”. See [Iida 2013].
We suppose quantity phrases which express non-proportional quantification constitute a special class of adjectival nouns\(^5\). Applying this axiom to the phrase “go-nin no gakusei”, we get the following.

\[
\text{(24) } \text{Val}(X, \text{“(go-nin no (gakusei))”} ) \iff \text{Val}(X, \text{“go-nin”} ) \land \text{Val}(X, \text{“gakusei”}).
\]

Suppose Q is a monotone increasing quantity phrase like “go-nin” and CN is a common noun. For a Japanese sentence of the form “((Q no (CN)) ga) V”, we have a following semantic axiom, which is one of the three subcases of the axiom about predication.

**Axiom of predication (monotone increasing case)**

If Q is monotone increasing, then \(\text{Val}(t, \text{“((Q no (CN)) ga) V”}) \) if and only if

\[
\exists X [\text{Val}(X, \text{“(Q no (CN))”}) \land \text{Val}(X, V)].
\]

Here \(t\) is a truth value true, and if \(S\) is a sentence, we usually write “\(S\) is true” instead of “Val\((t, S))\)”.

Given the previous (24), if we apply this axiom to (23), then it follows that the necessary and sufficient condition for (23) to be true is this.

\[
\text{(25) } \exists X [(\text{Val}(X, \text{“go-nin”}) ) \land \text{Val}(X, \text{“gakusei”} ) \land \text{Val}(X, \text{“waratta”})].
\]

The meanings of three Japanese words in (25) are given by lexical axioms like the one for “gakusei” which we already encountered at the end of the previous chapter\(^6\). The final outcome is something like the following.

\(^5\) Other kinds of adjectival nouns can be seen in the following examples.

- (a) orenji-iro no kuruma
  - orange color
  - car
  - (orange-colored car/cars)
- (b) daen-kei no tsukue
  - oval shape
  - desk
  - (oval-shaped desk/desks)
- (c) kinzoku-sei no kan
  - metal made
  - can
  - (a can/cans made of metal)

You can check that the above axiom is valid for these examples, too. Incidentally, it is interesting to note that quantity nouns are among the adjectival nouns which refer to colors ((a)), shapes ((b)) or material ((c)). As a matter of fact, the appearance of plural logic has prompted a revaluation of a thesis that number words work as adjectives and express numerical properties of objects, which many thought Frege had refuted in the 19th century. See [YI 1999].

\(^6\) The meaning of “go-nin” is given by the following axiom.
(23) is true if and only if there are some things which are five in number, students, and laughed\(^7\).

We should note that (25) is true when there are at least five students who laughed. This means that “Go-nin no gakusei” in (19) is interpreted as meaning “at least five students” according to our account\(^8\). If we are inclined to think that there are exactly five students who laughed when we hear the utterance of (23), then it can be explained as coming from its conversational implicature and not from its truth condition. If we wish to say that there are exactly five students who laughed, then we should use a non-monotone quantity phrase like “choudo go-nin” as in the following sentence.

(26) Choudo go-nin no gakusei ga waratta.

(exactly five-CL GEN student NOM laughed)

(Exactly five/The exactly five students laughed.)

Before tackling this, we had better look at a sentence with a monotone decreasing quantity phrase. “Shou-ssu” (small-number) that appears in (20) expresses a monotone decreasing non-proportional quantifier. Let us consider the following sentence.

(27) Shou-ssu no gakusei ga waratta.

(small-number GEN student NOM laughed)

(A/The small number of students laughed.)

\[\text{Val}(X, \text{“go-nin“}) \iff X \text{ are five.}\]

Under the policy adopted in this paper, this axiom is a special case of the following axiomatic scheme. See note 3.

Suppose CL is a classifier, then

\[\text{Val}(X, \text{“go-CL“}) \iff X \text{ are five.}\]

\(^7\) If we express the same in a Japanese metalanguage, (23)’s truth condition runs (“COP” is for copula):

Go-nin de-atte, gakusei de-atte, waratta mono ga aru.

five CL COP student COP laughed thing NOM exist

\(^8\) An indefinite noun phrase without any quantity phrase as in

(i) Gakusei ga waratta.

student(s) NOM laughed

(A/Some student/students laughed.)

might be classified as monotone increasing. “Gakusei” in (i) means the same as “at least one student”. Hence, our axiom of predication for monotone increasing case also applies to an indefinite noun phrase. For example, (i) is true if and only if

\[\exists X [\text{Val}(X, \text{“gakusei“}) \land \text{Val}(X, \text{“waratta“})].\]
It has also two interpretations, indefinite and definite, but here we consider only an indefinite one. It is clear that its truth condition cannot be given by a formula similar to (22) above.

(28) \( \exists X \ [A\text{-Small-Number-Of } (X) \land Students (X) \land Laughed (X)] \)

For, even when there are many students who laughed, (28) is true because we can take a few of them as witnesses to make (28) true.

One simple way of giving the truth condition of (27) is to think that it is the same as that of the negation of the following.

(28) Ta-su
no gakusei ga waratta.
many-number GEN student NOM laughed

(Many students laughed\(^9\).)

“Ta-su" (large number) is a contrary to “syou-su" (small number). For each monotone decreasing quantity phrase \( Q \), there is its contrary \( \overline{Q} \), which is monotone increasing. Thus\(^{10}\)

\[
\begin{align*}
\text{syou-su} & = \text{ta-su} \\
\text{gonin yori shou-su} & = \text{gonin ijyou}
\end{align*}
\]

With this notation, we can give the second subcase of the axiom of predication.

**Axiom of predication (monotone decreasing case)**

If \( Q \) is monotone decreasing, then a sentence \( "((Q \text{ no (CN)}) \text{ ga}) V" \) is true if and only if

\[
\forall X \ [(\text{Val}(X, V) \rightarrow \neg \text{Val}(X, "(\overline{Q} \text{ no (CN)})"))]
\]

From this axiom and the previous one for monotone increasing case, we derive the following as the truth condition of (27).

(29) If some things are students and laughed, then they are not many.

It might be objected that (29) expresses only a part of the truth condition of (27). For, from the utterance of (27) we infer that there are some students who laughed, but (29) does not imply this.

---

\(^9\) (28) can also be interpreted as having a definite subject, but this interpretation is irrelevant here.

\(^{10}\) “gonin yori shousuu” means less than five persons, and “gonin ijyou” means more than four persons.

19
For this objection, there are two possible responses. One is to modify the above axiom and make the existence of some CNs which V as part of the truth condition. Another is to argue that the existence claim is not part of the truth condition but a some sort of implicature. For example, consider the following.

(30) Gon-nin yori shou-suu no gakusei ga

waratta.

(Less than six students laughed.)

Is this true, when there were no students who laughed? It seems to me the answer is positive. If no students laughed, it is of course true that the number of students who laughed are less than six, and this seems to be enough to make (30) true. It is misleading to claim (27) when you know that there were none who laughed. Though such a claim constitutes a violation of some conversational maxim, the content of the claim will not be made false by it. Hence, I stick to a simpler axiom for the time being, and suppose that the existence claim is not part of the truth condition.

Now it is obvious how the truth condition of (26) should be given. It is the same as that expressed by the conjunction of (19) and (30). But we prefer the slightly different formulation that has the same content. A general pattern is given in the following axiom, which gives the third subcase of the predication axiom.

Axiom of predication (non-monotone case)

If Q is non-monotone, then a sentence “((Q no (CN)) ga) V” is true if and only if

∃X [Val(X, “(Q no (CN))“) ∧ Val(X, V) ∧ ∀Y [[Val(Y, CN) ∧ Val(Y, V)] → XηY]

A simple explanation might suffice. The truth condition of a non-monotone sentence consists of two parts connected by the same plural variable that is existentially quantified. The first part states that there are some things X that satisfy CN and V, and are at least as many as Q, and the second part states that if any objects also satisfy CN and V then X are found among them, namely, X constitute, as it were, “the smallest” group of the objects which satisfy CN and V.

3.1.2 Pattern (I) with a definite subject

So far, we have considered quantification of pattern (I) when its subject NP “Q + no + N” is indefinite. But, every sentence we have considered also has another
interpretation in which the subject NP is definite. For example, the sentence
(19) has a definite subject if it appears in a discourse like the following.

(31a) Sensei ga san-nin to gakusei ga
go-nin ita.
five CL were there

(31b) Go-nin no gakusei ga waratta.

(= (19)) five CL GEN student NOM laughed

(Three teachers and five students were there. The five students
laughed.)

In this occurrence of “go-nin no gakusei” it refers to certain five students
who are introduced into the discourse by (31a). This expression may be replaced
by “sono go-nin no gakusei”. In general, if “sono” precedes a noun phrase NP,
the resulting phrase means something like “that/those NP” or “the NP”. As is
shown above, an appropriate translation of “go-nin no gakusei” in (31b) is “the
five students”.

Then, “go-nin no gakusei” in (31b) corresponds to an English definite (plural)
description. If we extend a classical analysis of definite singular descriptions
that originated in Russell to plural cases, then a definite plural description “the
αs” would refer uniquely to a certain individuals that satisfy the conditions
expressed by α.

Such an analysis, however, does not seem to work here. If we adopt it,
“go-nin no gakusei” in (31b) should uniquely refer to those individuals that are
students and five in number. If it is understood in the context that there are
more than five students in the domain of the discourse, then there are various
possibilities of selecting five students, and there cannot be any unique group
of five students that can be described as “the five students”. On the other
hand, if it is understood that there are just five students in the domain, then
the quantity word “go-nin” would not have any semantical contribution and
“gakusei”(students) by itself would succeed in securing the reference11.

The difficulty becomes more obvious, if we consider a case in which monotone
increasing “go-nin” is replaced by monotone decreasing “go-nin ika” (less than
six persons). If there are more than one student, then there are more than one
way of selecting the students so that they meet the condition expressed by a
plural description “go-nin ika no gakusei” (less than six students), and it would
never be correct to speak of the definite “go-nin ika no gakusei” (the less than six

11 If there are less than five students, then the description would be empty and hence (31b)
would be false or lack a truth value, depending on what account of definite description you
adopt. We are going to discuss the relating issues in Chapter 6. By the way, note that it
seems natural to think that the antecedent of the previous sentence, which is a conditional, is
ture when there are no students. This seems to give a support for our version of predication
axiom for a monotone decreasing case.
students). But, the above two sentences in (31) are perfectly understandable, even if we replace the two occurrences of “go-nin” with those of “go-nin ika”.

A natural solution to this is to construe (31b) as having a similar structure as an English sentence with a non-restrictive relative clause, say, “The students, who are five in number, laughed”. Thus, “go-nin no” (of five persons) is not a part of the definite description, but a predicate that is applied to the students who were introduced in the previous (31a). The fact that we can paraphrase “go-nin no gakusei” in (31b) as “go-nin no sono gakusei” (five those students) as well as “sono go-nin no gakusei” (those five students) might be an evidence for such a reading.

In the present example, the description “go-nin no gakusei” (the five students) or “go-nin ika no gakusei” (the less than six students) is working anaphorically, referring back to “gakusei” that occurs in the previous sentence. Let us call such a use of a definite description as an “anaphoric use”.

In sum, if a quantity phrase which expresses a non-proportional quantifier occurs in an anaphoric use of a definite description, it is not really a part of the description, but it adds a clause in which a quantity phrase functions as a nominal predicate.

There are cases in which a plural definite description with a quantity phrase refers to a certain group of the objects without any help from anaphoric relation. Consider the following.

(32) Taro ga oshieta go-nin no gakusei
    NOM taught five CL GEN student
    ga atsumatta.
    NOM gathered

(The five students whom Taro had taught gathered.)

If a speaker and her audience know Taro, then their conversation might start with an assertion of (32). Though (32) can be interpreted as saying that five of the more than five students Taro had taught came together, it is not the only interpretation. As the above translation shows, it might also mean that all the five students Taro had taught came together.

In such a case, Russellian analysis seems to be appropriate, namely, “Taro ga oshieta go-nin no gakusei” (the five students whom Taro had taught) refers uniquely to a group of objects which are five, students, and had been taught by Taro. Let us call such a use of a definite description as an “attributive use”. But, even in this case, the numerical qualification “go-nin no” (being five persons) has no part in picking out what the entire noun phrase refers to. For, on one hand, if the definition succeeds in referring to a number of objects, then there must be exactly five students whom Taro had taught and they are what “Taro ga oshieta gakusei” (the students Taro had taught) refers to; on the other, if Taro had taught more than or less than five students, then the entire noun
phrase “Taro ga oshieta go-nin no gakusei” (the five students whom Taro had taught) fails to refer to any object.

Hence, if a quantity phrase Q occurs in a definite NP of the pattern “Q + no + N”, Q does not contribute to identify the reference of the NP, whether the NP is construed as referential or attributive.

And, in this case also, a solution seems to be to construe “go-nin no” not as part of the definite description but an additional predicate. Then, there seem to be two assertions which are involved in (32), namely,

1. that all the students Taro had taught gathered, and
2. that the students Taro had taught are five in all.

If it turns out that Taro had taught more than or less than five students altogether, then 2 is false. In this case, will (32) be also false?

Of course, this is an option we may take. But, it is not the only option. Somewhat surprisingly, there are two other options which might be defended as well as this one. We might argue that (32) will be still true. Or, we might argue that (32) will be neither true nor false. For both of these options, it is essential to hold that, even though (32) may involve two assertions, they are not of the same importance.

You may argue that (32) can be true even when the total number of the students Taro had taught are not five. It is because what is primarily asserted by (32) is that all the students Taro had taught gathered, and the claim that they are five in number is only of a subsidiary issue and its falsity does not make the entire statement false.

According to this view, even though the quantitative condition expressed by Q is not satisfied by the objects referred to by the entire NP, this by itself may not make the sentence in which it occurs untrue.

A similar consideration applies to the cases in which a definite description is used not attributively, but anaphorically. Suppose that there are a number of students who are drinking beer in a corner, and my friend says that

(33) Gakusei ga go-nin sumi ni iru.

(Five students are in the corner.)

Further suppose that I respond to her, saying that

(34) Go-nin no gakusei wa doukyunsei-da.

(The five students are classmates.)

\[12\] What happens if we replace “go-nin” with “go-nin ika” (less than six persons) in (32)? Even in such a case, “go-nin ika no gakusei” must refer to a certain determinate group of students who are less than six in number.

\[13\] This example is not the same as, but closely related to a “referential” use of a definite description ([Donnellan 1966]).

23
But, my friend miscounted the number of the students in the corner, and there are only four students there. Does her miscount make my assertion of (34) untrue? Here the intuition that (34) is still true as long as all the students in the corner are classmates seems to be strong.

The last option is to hold that in similar cases the statements in question are neither true nor false. In this option also, the condition that is expressed by a quantity phrase is a subsidiary one, but now it is regarded as a presupposition for the entire statement.

This option appears attractive if you think that, in a situation like the one which is in consideration here, it is neither correct to agree with the statement, nor dissent from it. The acceptance of a truth value gap, however, forces us to revise the classical logic, which is a bigger divergence from the standard logic than adopting plural logic. Our plural logic is classical plural logic, and it is an extension of classical logic, not its revision. For this reason, many philosophers are reluctant to introduce truth value gap into semantics, but there is no denying that there are many phenomena in natural language which cry for introducing truth value gap and many linguists do not hesitate to do that.

It is not easy to decide which of the three options is the best one. Fortunately, however, it is not the task which we should discharge right now, either.

The main conclusion of this part of our discussion is as follows.

In both anaphoric and attributive uses of a definite description, what the entire phrase refers to may be determined without any contribution from the quantity phrase occurring in it. The quantity phrase ascribes a certain numerical property to what the the entire phrase refers to, and this ascription should be regarded as a subsidiary comment on the principal content which the sentence expresses.

Thus, a non-proportional quantifier expressing quantity phrase that occurs in a definite noun phrase works as a predicate ascribing a numerical property to a group of objects, and hence, has the semantic function which is the same as that of its use as a nominal predicate. Hence, for its account, we can safely refer to our discussion of a quantity phrase as a nominal predicate in §3.3.

### 3.1.3 Pattern (II)

Pattern (II) represented by

\[(20)\] Gakusei shou-suu ga waratta.

A/The small number of students laughed.

---

14 We are going to accept truth value gaps with regard to indeterminate phrase quantification, which is our chief subject here. Hence, you will hear a lot about presupposition and truth value gaps later.
does not offer anything new in the matter of semantics. A subject NP may be indefinite or definite, and if a subject NP is indefinite, there are three different ways to interpret the semantic import of predication depending on whether the subject NP is monotone increasing, monotone decreasing, or non-monotone. We can apply the above Axiom of Predication with its three subcases to this pattern with just minor modification. Only one new axiom is necessary, and it is about the semantics of an NP consisting of a common noun CN and a quantity phrase Q expressing a non-proportional quantifier.

**Axiom of postpositioned quantifier phrase**

\[
\text{Val}(X, (\text{CN Q})) \iff \text{Val}(X, \text{CN}) \land \text{Val}(X, \text{Q})
\]

It might be argued that a construction shown in the following example comes from Pattern (II).

(35) Shou-su˚ ga waratta.
    small number NOM laughed
    (A/The small number (of people) laughed.)

Given a suitable context, (35) is perfectly grammatical and can be easily understood. In general, Pattern (II) have a slightly different variety

[(II') Q + ga + V.]

This means that when the domain of a quantity phrase is contextually understood, it need not be explicitly stated. This is another example of the strong tendency in Japanese to avoid explicit mentioning of the things understood in the context.

### 3.1.4 Pattern (III)

There are two distinctive features in Pattern (III). First, a quantity phrase Q appears within a verb phrase as an adverbial in this pattern. Secondly, a subject NP which appears in this pattern admits only an indefinite interpretation\(^{15}\). I begin with the second feature.

There is a test to see whether a given occurrence of an NP is an indefinite one or not in Japanese\(^ {16}\). It is to see whether a sentence ending with “aru” or “iru” derived from the original sentence can be interpreted as an existential statement. As an example, consider (19), which I repeat here.

---

\(^{15}\) Masuoka and Takubo also note that Pattern (III) is found mainly with indefinite NPs. See [Masuoka and Takubo 1993, p.98].

\(^{16}\) See [Iida 2007].
From this, we can form another sentence ending with “iru”

(36) Waratta go-nin no gakusei ga iru.
    laughed five-CL GEN student NOM are

(36) is ambiguous. It may mean either (a) there are five students who laughed, or (b) the five students who laughed are at some place that is understood in the context. Let us call the former an existential interpretation, and the latter a locative interpretation.

We should remind us that (19) itself is ambiguous between an indefinite reading and a definite reading as its English translation shows. The important thing is that we can distinguish two readings of (19) by seeing whether it can be transformed to (36) without changing its truth condition. An indefinite reading of (19) can have the same truth condition as (36) in its existential reading, while a definite reading of (19) cannot have the same truth condition as (36) in either reading.

Let us try this test to a Pattern (III) sentence (21) shown below.

(21) Gakusei ga chyoudo san-nin waratta.
    student NOM exactly three CL laughed

(21) has the same truth condition as the reading (a) of (37). Moreover, in contrast to (19), it is hard to get a definite reading of (21), that is, a reading according to which “gakusei” refers to some definite group of students.

In general, a subject NP in Pattern (III) is indefinite. But here I must add an important qualification, which is necessary because of the existence of sentences like the following.

(38) Gakusei wa chyoudo san-nin waratta.
    student TOP exactly three CL laughed
In this sentence, “gakusei” should be definite because it is the topic of the sentence. (38) would be translated into English as “Exactly three of the students laughed”, and hence, its meaning is different from (21).

It is very important to realize there exists a sentence which has a definite noun phrase as its subject and a quantity phrase occurs adverbially in its predicate. It is also important, however, to notice that such a sentence does not have Pattern (III) because of the presence of the topic maker “wa”. (38) has “wa” instead of “ga” unlike (21), and this makes a big difference. We are going to be preoccupied with an analysis of sentences like (38) in Chapter 6.

Another distinctive feature of Pattern (III) is that a quantity phrase functions as an adverbial in a sentence of this pattern. In contrast to most adverbials, which give information about the modes of events or states described by main verbs, quantity phrases occurring in Pattern (III) sentences give information about the quantities of the objects that participate in those events or states. If we compare (21) with the following (39), the difference will become clear.

(39) Gakusei ga oo-goe de waratta.

Students/The students laughed loudly.

In (39) an adverbial phrase “oo-goe de” (with loud voice) tells us the mode of students’ laughing, but in (21) “choudo san-nin” (exactly three) is not concerned with the mode of laughing but the number of its agents. If we adopt a Davidsonian analysis of action sentences, then an adverbial like “oo-goe de” is taken to be a predicate that is true of an event that is introduced by the verb “waratta”. In contrast, “choudo san-nin” in (21) may be taken as a predicate that is true of the objects which fill the agent argument of the verb

Just as Pattern (I) with an indefinite subject, there are three different cases to consider in Pattern (III), namely, three cases in which the relevant quantity phrase Q is monotone increasing, monotone decreasing, or non-monotone.

I present a semantic axiom for Pattern (III) sentences in a very limited way. It is limited in at least two respects. First, I consider simple sentences having a one-place, or intransitive, verb as its only predicate, as I have been doing for other cases in this sketch. Secondly, an adverbial quantity phrase Q can occur at a place in a sentence other than the place just after a noun phrase which it modifies as in the following example.

---

17 As this paper is concerned with semantics of noun phrases, we are not going to do “subatomic semantics” (see [Parsons 1994]), that is, to analyze Japanese verbs as having event arguments as well as agent or theme arguments. Our treatment of Japanese verbs in this paper is “shallow” and their semantic values are those objects that play the roles of agents, themes and the like. Of course, in the fuller semantic treatment of the sentences we are considering here, we should go “deeper” into the structure of Japanese verbs.

27
Chyoudo san-nin Gakusei ga waratta.

Exactly three CL student NOM laughed

(Exactly three students laughed.)

The first limitation seems to be serious, because it means that our account does not cover the cases of multiple quantification, the cases in which several possibly different sorts of quantification occur in one sentence. These limitations can be overcome, however, with a number of more complex axioms of predication which cover different types of predicates. For example, Axiom of Predication we encountered above is formulated for only one type of predicates, namely, intransitive verbs, and it has three subcases depending on the kinds of NP; it should be reformulated so that it covers more types of predicates, and for each new type of predicate, we have three subcases just as before. Although this seems to be very complicated, it is not really so, because we can use the same pattern in each of different predicate types. The complication is only that of a book-keeping sort. As a matter of fact, the present axiom can be extended to monadic (one-place) predicates in general without any difficulty. And we don’t need to consider any predicate which is more complex than a three-place one.

As for the second limitation, this is just because I have not yet done a necessary work to collect the data about the distributions of adverbial quantity phrase in a sentence. I believe, however, it would not be so difficult to have a satisfactory account of adverbial quantity phrases, once we have a precise and exhaustive characterization of the relevant data.

For this limited class of cases, our semantics for Pattern (III) sentences is this. You will see at once it is not much different from the Axiom of Predication we set down for Pattern (I) sentences with indefinite subjects.

**Axiom of a quantity phrase as an adverbial**

Let $S$ be a sentence of the form “(CN ga)(Q V)”, where CN is a common noun, Q is a quantity phrase that expresses a non-proportional quantifier, and V is an intransitive verb.

(i) If Q is monotone increasing, then $S$ is true if and only if

$$\exists X [\text{Val}(X, \text{CN}) \land \text{Val}(X, (Q V))]$$

(ii) If Q is monotone decreasing, then $S$ is true if and only if

$$\forall X [\text{Val}(X, \text{CN}) \rightarrow \neg \text{Val}(X, (Q V))]$$

(iii) If Q is non-monotone, then $S$ is true if and only if

$$\exists X [\text{Val}(X, \text{CN}) \land \text{Val}(X, (Q V)) \land \\
 \forall Y [[\text{Val}(Y, \text{CN}) \land \text{Val}(Y, V)] \rightarrow X \eta Y]]$$

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There is one thing missing for deriving the truth condition of a sentence of Pattern (III) from this axiom. It is this.

**Axiom of adverbial quantity modification**

Let Q be a quantity phrase that expresses a non-proportional quantifier and V an intransitive verb. Then

\[ \text{Val}(X, (Q \ V)) \Leftrightarrow \text{Val}(X, Q) \land \text{Val}(X, V) \]

As I remarked before, generally an adverbial phrase modifies the verb it attaches by functioning as an additional predicate true of the events or states which are introduced by the verb. An adverbial quantity phrase is an exception to this general rule, and it works as an additional predicate true of one of the arguments which the verb takes such as an agent argument and a theme argument.

### 3.2 Quantity phrases that function as proportional quantifiers

Quantity phrases like “subete” (all), “dai-bubun” (large part), “hanbun” (half), and “ni-jyuu paasento” (twenty percent) are different from those quantity phrases we have been considering in that they are concerned with a certain totality of things and say what is the proportion of the number of things among them that satisfy a property that is given by the predicate part of the sentence. For example, consider the following sentence.

(41) Dai-bubun no gakusei ga waratta.

(A large part of the students laughed.)

This sentence says that the number of the students who laughed is much higher than that of half of all the students.

At first sight, it might be thought that it is possible to give the truth condition of (41) in a way that we have been doing for other quantity phrases. In particular, some might think the following gives the truth condition of (41).

(42) \[ \exists X \left[ \text{Val}(X, \text{“dai-bubun no gakusei”}) \land \text{Val}(X, \text{“waratta”}) \right] \]

According to this, (41) would be true if and only if there are some people X who are *dai-bubun no gakusei* (a large part of the students) and laughed. But, how can we know that a given X satisfies the first conjunct? We might know the number of X, but how do we know that X are a large part of the students? In
order to be able to do that, we have to know the total number of the students, and compare it to the number of students who laughed.

Here is an essential difference between a proportional quantifier and a non-proportional quantifier like “go-nin” in a sentence like (19) “Go-nin no gakusei ga waratta” (Five students laughed). In the latter, it suffices to see whether there are at least five students who laughed among the students in question and there is no need to compare them to the totality of the students.

In a sentence like (41), we have to consider a certain totality of things and some things among them which satisfy an additional condition. In (41), they are the totality of the students and those among them who laughed. Both of them are given by definite plural descriptions. In this case, they are “gakusei” (the students) and “waratta gakusei” (the students who laughed); although it is not explicit in Japanese, they are both definite NPs as their English translations indicate. In fact, we can paraphrase (41) in the following way (“COP” is for copula).

(43) Waratta gakusei wa gakusei no dai-bubun da.

(The students who laughed are a large part of the students.)

(41) has the form “Q + no + N + V”, and hence, is of Pattern (I) of §1.3.1. Besides the three patterns mentioned there, there is another pattern for quantity phrases that express proportional quantifier phrase to appear in a sentence. Here is an example.

(44) Gakusei no dai-bubun ga waratta.

(A large part of the students laughed.)

Let us call such a pattern as Pattern (IV).

(IV) N + no + Q + ga + V.

An interesting fact is that quantity phrases “ta-suu” (large number) and “shou-suu” (small number) are ambiguous; they express non-proportional quantifiers when they occur in Patterns (I)–(III), while they express proportional quantifiers when they occur in Pattern (IV). This becomes obvious if we compare the following two sentences.

(45) Ta-suu no gakusei ga waratta.

(Many students laughed.)

(46) Gakusei no ta-suu ga waratta.

(Many of the students laughed.)
"Ta-su" in (45) expresses a non-proportional quantifier, while the same word in (46) expresses a proportional quantifier. It is just as it is the case with English "many."\(^{18}\)

It is important to note that “ta-su no gakusei” in (45) might be interpreted as indefinite, but that “gakusei no ta-su” in (46) can never be interpreted as indefinite. This can be seen by applying the “test” for indefiniteness that I mentioned above. If we transform (45) and (46) into sentences ending with “iru”, the results are these.

\(^{18}\) A natural question that occurs here is whether a non-proportional quantifier Q also can show Pattern (IV). Consider the following.

(i) ?Gakusei no san-nin ga kita.
   student GEN three-CL NOM came.
   (Supposedly: The three of the students came.)

(ii) ?Gakusei no san-nin ika ga kita.
    student GEN three-CL less than NOM came.
    (Supposedly: Less than three of the students came.)

(iii) ?Gakusei no choudo san-nin ga kita.
     student GEN just three-CL NOM came.
     (Supposedly: Just three of the students came.)

A more natural way of saying what these sentences try to express is to use the expression like “gakusei no uchi no san-nin” (three among the students). Hence, the possible pattern is not Pattern (IV), but the following pattern

(IV') \(N + no + uchi + no + Q + ga + V\).

Interestingly, a proportional quantifier also exhibits this pattern as the following example shows.

(iv) Gakusei no uchi no hotondo ga kita.
    student GEN within GEN most NOM came.
    (Most of the students came.)

It is also important to note that if we put the topic marker right after “gakusei” instead of “no” in (i)–(iii), they become perfectly grammatical and express intended meanings.

(i') Gakusei wa san-nin ga kita.
    student TOP three-CL NOM came.
    (The three of the students came.)

(ii') Gakusei wa san-nin ika ga kita.
     student TOP three-CL less than NOM came.
     (Less than three of the students came.)

(iii') Gakusei wa choudo san-nin ga kita.
      student TOP just three-CL NOM came.
      (Just three of the students came.)

This type of sentences will be discussed in a great detail in Chapter 6.

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While (45') may have an existential reading according to which it says that there are a large number of students who laughed, the only possible reading of (46') is a locative one, according to which it says that a large number of the students who laughed are here (or at a certain place that is understood in the context) now.

This clearly shows that “gakusei no ta-su” in (46) is a definite NP. It does not mean, however, that “gakusei no ta-su” is a definite (plural) description. For, it is obvious there are various groups of students who can be said to constitute a large part of the students. The definiteness comes from some other source.

We said that there are four patterns for a proportional quantifier quantity phrase $Q$ to appear in a sentence. In reality, there is another pattern which we already met before in (43). This pattern, which we call Pattern (V), has $Q$ as part of a nominal predicate.

\[
\text{(V) } V + N + wa + N + no + Q + da.
\]

Every sentence of Patterns (I)–(IV) can be rewritten as a sentence of this pattern without changing its truth condition. Thus, we can regard Pattern (V) as a sort of canonical form for a proportional quantifier sentence.

---

\[19\] In general, our test for indefiniteness yields the same conclusion for any sentence with a proportional quantifier quantity phrase. Look at the four sentences (1)–(4) at the very beginning of this paper. They exemplify the four patterns for a proportional quantifier quantity phrase to appear in a sentence. You can see that the following four sentences have only locative reading, and never existential readings.

\[
\begin{align*}
(1') & \text{ Kita subete no gakusei ga iru.} \\
& \text{came all GEN student NOM are.} \\
& \text{(All the students who came are here.)} \\
(2') & \text{Kita gakusei no subete ga iru.} \\
& \text{came student GEN all NOM are.} \\
& \text{(All of the students who came are here.)} \\
(3') & \text{Kita gakusei subete ga iru.} \\
& \text{came student all NOM are.} \\
& \text{(All the students who came are here.)} \\
(4') & \text{Kita gakusei ga subete iru.} \\
& \text{came student NOM all came.} \\
& \text{(The students who came are all here.)}
\end{align*}
\]
My account of quantity phrases expressing proportional quantifiers is based on two assumptions about the “logical form” of a sentence of this canonical pattern\textsuperscript{20}.

1. Two NPs occurring in (V) are definite plural descriptions.

2. A quantity phrase Q expressing a proportional quantifier is a two place plural predicate.

This presupposes that we have an account of plural definite descriptions. We adopt a Russelian account of them. Consider the following sentence with a reading that is indicated by its English translation.

\[(49) \text{Gakusei ga atsumatta} \]
\[
\text{student(s) NOM assembled}
\]
\[
(\text{The students assembled.})
\]

We suppose that the truth condition of (49) is given by the following formula in plural logic\textsuperscript{21}.

\[(50) \exists X [\text{Val}(X, \text{“gakusei”}) \land \forall Y[Y \eta X \leftrightarrow \text{Val}(Y, \text{“gakusei”})]]
\land \text{Val}(Y, \text{“atsumatta”})]
\]

There are three conjuncts inside the existential quantifier. The first two conjuncts may be regarded as a complex predicate with “X” free that corresponds to the plural definite description “gakusei”, and says that X are the things (people) which constitute the totality of students under the context.

As it will be handy to have a shorthand notation for this predicate, let us adopt the following convention.

**A notation for a plural definite description**

Let \( \Phi \) be a predicate in the metalanguage with “X” free. Then, let

\[\triangle(X, \Phi)\]

be an abbreviation of

\[\Phi(X) \land \forall Y[\eta YX \leftrightarrow \Phi(Y)].\]

When \( \Phi(X) \) is Val(X, \( \alpha \)) where \( \alpha \) is a plural definite description, “\( \triangle(X, \Phi) \)” is written as

\[\triangle(X, \alpha)\]

and read “X are the \( \alpha \)”, or in Japanese, “X wa \( \alpha \) da” provided that “\( \alpha \)” is understood as definite.

\textsuperscript{20} They are also the basic assumptions of an account of generalized quantifiers ([Rayo 2002]) or non-proportional quantifiers ([McKay 2006, pp.61ff.]) in plural logic.

\textsuperscript{21} Cf. [McKay 2006, p.165].
With this, (50) can be rewritten as

\[(51) \exists X[\triangle(X, \text{“gakusei”}) \land \text{Val}(X, \text{“atsumatta”})]\]

Now we can give a simple version of an axiom for a quantity phrase that function as a proportional quantifier. Although there may be various differences between Pattern (I)–(IV), they have the same truth condition at least in simple cases.

**Axiom of a proportional quantifier quantity phrase**

Let \(S\) be a sentence of one of the following forms

\[
\begin{align*}
(Q \text{ no } CN \text{ ga}) & \ V \\
(CN \text{ no } Q \text{ ga}) & \ V \\
(CN \text{ no } Q \text{ ga}) & \ V \\
(CN \text{ ga}) & \ (V \ V),
\end{align*}
\]

where \(CN\) is a common noun, \(Q\) is a quantity phrase that expresses a proportional quantifier, and \(V\) is an intransitive verb. Then

\[
S \text{ is true } \iff \exists X \exists Y[\exists X[Y[\triangle(X, (V \ CN)) \land \triangle(Y, \ CN)] \land \text{Val}((X,Y), Q)]].
\]

Thus, the truth condition of (41) is given by the following formula in our metalanguage at the first stage.

\[(52) \exists X \exists Y[\exists X[Y[\triangle(X, (\text{waratta gakusei})) \land \triangle(Y, \text{gakusei})] \land \text{Val}((X,Y), \text{dai-bubun})]],
\]

which is an abbreviation of a more complex formula. Of course, we need an axiom which gives us the meaning of the quantity phrase “dai-bubun”. This can be done in a straightforward way if our metalanguage is Japanese.

\[
\text{Val}((X,Y), \text{“dai-bubun”}) \iff X \text{ wa } Y \text{ no dai-bubun da}.
\]

Then, remembering that “\(\triangle(X, \alpha)\)” may be read in Japanese “\(X \text{ wa } \alpha \text{ da}\)”, we can derive from (52) and this axiom (and some logic)

\[(53) \text{Waratta gakusei wa gakusei no dai-bubun da},
\]

which is the same as (43) above.
3.3 Quantity phrases are first-order plural predicates

Although it may seem that our account of quantity phrases that express proportional quantifiers is much different from that we gave for those that express non-proportional quantifiers, there is a deep similarity between them. It is that a quantity phrase is a plural predicate whether it expresses a non-proportional quantifier or a proportional one. To see this, it is enough to compare the two lexical axioms, one of which is a non-proportional quantifier and the other is for a proportional quantifier.

\[(A) \text{Val}(X, "ta-suu") \iff X \text{ wa ta-suu da (}X\text{ are many)}.\]

\[(B) \text{Val}(\langle X, Y \rangle, "ta-suu") \iff X \text{ wa } Y \text{ no ta-suu da (}X\text{ are many of }Y).\]

As we have seen, a quantity phrase “ta-suu” (many number) is ambiguous. It may express a non-proportional quantifier as in (45), or a proportional quantifier as in (46). Axiom (A) is for a non-proportional case, and Axiom (B) is for a proportional case. They show that the ambiguity lies in whether it is a one-place predicate or a two-place one.

It is also important to note that both are plural predicates that are true of individuals. They are first-order predicates, and not some higher-order predicates, like second-order predicates that are applicable to first-order predicates as in Frege, or predicates that represent a certain set of sets as in generalized quantifier theory.

In general, Japanese quantity phrases are first-order plural predicates of which some are one-place predicates expressing non-proportional quantifiers, and the rest are two-place predicates expressing proportional quantifiers.

This can be seen from the fact that a quantity phrase may form a nominal predicate. As in the canonical pattern for a proportional quantifier

\[(V) V + N + wa + N + no + Q + da.\]

A quantity phrase Q which expresses a non-proportional quantifier can appear as a nominal predicate in the following pattern.

\[(VI) V + N + wa + Q + da.\]

Sentence which correspond to (19)–(21) and exemplify this pattern are these.

\[(54) \text{Waratta gakusei wa go-nin da.}\]

laughed student TOP five-CL COP

(There are five students who laughed / The students who laughed are five in number.)
As English translations indicate, a quantity phrase in each sentence can be interpreted as either indefinite or definite. A semantic account we gave for a sentence of Pattern (I) can be adopted to give the truth conditions of (54)–(56), whether it has an indefinite reading or a definite one.

In fact, Pattern (VI) is a special case of a more general pattern

(VI') NP + wa + Q + da.

This includes the sentences like these.

(57) Gakusei wa choudo go-nin da.
    student TOP exactly five-CL COP
    (There are exactly five students / The students are exactly five
     in number.)

(58) Hanako ga oshieta gakusei wa jyu-nin ika
    NOM taught student TOP ten-CL less then
    da.
    COP
    (There are less then ten students whom Hanako taught / The students
     Hanako taught are less than ten in number.)

For indefinite cases of Pattern (VI') sentences, we may adopt the following axiom.

**Axiom for a non-proportional quantifier quantity phrase as nominal predicate (indefinite case)**

Suppose NP is an indefinite noun phrase and Q a non-proportional quantity phrase.

(i) If Q is monotone increasing, then a sentence “(NP wa) (Q da)” is true if and only if

\[ \exists X [\text{Val}(X, NP) \land \text{Val}(X, Q)] \]

(ii) If Q is monotone decreasing, then a sentence “(NP wa) (Q da)” is true if and only if
\[ \forall X[\text{Val}(X, \text{NP}) \rightarrow \neg \text{Val}(X, \text{Q})] \]

(iii) If Q is non-monotone, then a sentence “(NP wa) (Q da)” is true if and only if

\[ \forall X[\text{Val}(X, \text{NP}) \land \text{Val}(X, \text{Q})] \land \forall Y[\text{Val}(Y, \text{NP}) \rightarrow X \eta Y] \]

Moreover, Pattern (VI') is also seen with a proportional quantity phrase, as the following examples show.

(59) Waratta gakusei wa zen-in da.
    laughed student TOP all COP
    (All of the students laughed.)

(60) Gakusei wa hanbun ika dat-ta
    student TOP half less than COP-past
    (The students were less than half of the all.)

Such sentences can be regarded as an abbreviated form of a generalized version of Pattern (V). If the totality to which the part is to be compared is easily guessed at, then there is no need to explicitly mention the totality in question. With (59), the totality that is to be compared to the part is what “gakusei” refers to, namely, all the students. With (60), we should suppose the totality is somehow given in the context, say, an audience for a lecture.

Then, we can adopt our axiom of a proportional quantifier quantity phrase for a case when it appears as a nominal predicate in Pattern (VI').

**Axiom of a proportional quantifier quantity phrase as nominal predicate**

Suppose NP is a noun phrase and Q a proportional quantifier quantity phrase.

Then, a sentence “(NP wa) (Q da)” is true if and only if

\[ \exists X[\Delta(X, \text{NP}) \land \text{Val}(\langle X, A \rangle, \text{Q})] \]

where A is a certain totality given in the context.

The fact that a quantity phrase can be regarded as a first-order predicate is also important for a prospect of a more general theory of quantification in Japanese, which can treat mass quantification as well as count quantification we have been concerned with. We can easily imagine that there are one-place
predicates for mass quantification like “takusan” (much) and “san-kiro” (three kilograms) as well as two-place predicates for proportional mass quantification like “dai-bubun” (most) and “han-bun” (half). Many of these predicates for mass quantification have the same form as those for count quantification, and a unified theory of quantification should explain how it comes about. What is needed most for such a theory is a satisfactory account of mass predication, just as an account of plural predication has been essential for plural logic. But, it is a topic for another occasion.
Chapter 4

Quantification by “dono” phrase. Is “dono” a singularizing predicate?

4.1 Singular quantification and association with “mo”

As we said at the beginning of this inquiry, quantification by means of “dono” phrase is a special case of indeterminate phrase quantification. One of the most striking features of this kind of quantification is that many of its instances are singular quantification. It is all the more notable considering that Japanese is mostly number-neutral. Quantification by means of a “dono” phrase like (61) and (62) below gives us the clearest case of singular quantification in Japanese.

(61) Dono kodomo mo waratta.
child(ren) ∀ laughed

(Every child laughed.)

(62) Dono kodomo ka ga waratta.
child(ren) ∃ NOM laughed

(Some child laughed.)

In either of these, the quantification involved is singular. If we compare (61) with a sentence with a quantity phrase “subete” (all), the difference becomes even clearer.

(63) Subete no kodomo ga atsumatta.
all GEN child(ren) NOM gathered
(All children gathered.)

Other indeterminate phrases like “dare” (who) and “dore” (which) also make singular quantification. You can see this from the sentences (5) and (6), which I repeat here.

(5) Dare mo ga waratta.
who ∀ NOM laughed

(Everybody laughed.)

(6) Dare ka ga waratta.
who ∃ NOM laughed

(Somebody laughed.)

Another characteristic feature of indeterminate phrase quantification is that a quantifier particle “mo” or “ka” is essential to it. Conversely, if “mo” and “ka” appear without any indeterminate phrase, then they don’t have any quantificational force. In other words, singular quantification in Japanese is made possible by a combination of an indeterminate phrase and a quantifier particle.

Indeterminate phrase quantification with a universal quantifier “mo” has a complexity that does not exist when it appears with an existential quantifier “ka”.

An indeterminate noun phrase is either an indeterminate term like “dare” (who) and “dono hon” (which book), or a complex phrase like “dono kodomo no oya” (a parent/parents of which child) and “itsu kita hito” (a person who came when). An existential quantifier particle “ka” can be used only with an indeterminate term, as is the case with (6) and (62).

In contrast, there is no such restriction for a combination of an indeterminate phrase and “mo”. “Mo” need not be right after an indeterminate term; it may be located after an entire complex indeterminate phrase, and hence, away from indeterminate words like “dono” and “dare”. Here are some examples.

(64) Dono kodomo no oya mo kita.
child(ren) GEN parent(s) ∀ came

(Any child’s parent(s) came.)

(65) Dono kodomo ga kaita tegami mo todoita.
child(ren) NOM wrote letter(s) ∀ arrived

(Any letter(s) that any child wrote arrived.)

(66) Dare ga kaita tegami mo todoita.
who NOM wrote letter(s) ∀ arrived

(Any letter(s) that anybody wrote arrived.)

---

1 See Chapter 1.
If we put “ka” instead of “mo” in these sentences, then the results are grammatically doubtful at most.

?(67) Dono kodomo no oya ka ga kita.
      child(ren) GEN parent(s) ∃ NOM came
      (Supposedly: Some parent(s) of some child came.)

?(68) Dono kodomo ga kaita tegami ka ga todoita.
      child(ren) NOM wrote letter(s) ∃ NOM arrived
      (Supposedly: Some letter(s) which some child wrote arrived.)

?(69) Dare ga kaita tegami ka ga todoita.
      who NOM wrote letter(s) ∃ NOM arrived
      (Supposedly: Some letter(s) which somebody wrote arrived.)

From this brief survey of the relevant data, we can see that there are at least three questions any satisfactory account of indeterminate phrase quantification, in particular, quantification by “dono + N”, should answer.

1. Why is it a singular quantification?
2. Why are both an indeterminate phrase and a quantifier particle necessary for quantification?
3. Why may “mo” associate with an indeterminate phrase in a long-distance manner, while “ka” may not?

And, if we remember what we noted in Chapter 1 about a close relation between indeterminate phrase quantification and interrogatives as well as the fact that indeterminate words form a family with demonstrative words, we realize that there are two more questions which such an account should answer as well, namely,

4. Why is an indeterminate phrase used both in quantification and interrogative construction?
5. What do indeterminate words have in common with demonstrative words?

Hopefully, an account I am going to present will answer all of these questions.

4.2 Indeterminate NP with “dono + N”

Before trying to answer these questions, we should have a more precise characterization of an indeterminate phrase, in particular, an indeterminate noun phrase with “dono”.

First, there are indeterminate terms. They have the form
dono + N

with N a common noun. The examples are

\textit{(dono kodomo)} (which child)
\textit{(dono tegami)} (which letter)

In Japanese, there are two ways of forming a complex noun phrase. First, a noun phrase can be modified by another noun phrase by connecting them by a case particle “no”. Secondly, it can be modified by a modifying clause, which is logically a one-place predicate.

In order to have a concrete example to work with, let us suppose that our vocabulary consists of common nouns, verbs taking one or two arguments, an indeterminate word “dono”, quantifier particles “mo” and “ka”, and case particles “ga”, “o” and “no”. Let us call a fragment of Japanese built from such a vocabulary “J”.

The aim of constructing such a fragment is to show as concrete as possible how this part of the language works in a way that is, and it does not mean that our consideration in coming chapters will be restricted to this fragment.

In the following definition, a noun phrase and its subclass, an indeterminate noun phrase, are characterized simultaneously.

\textbf{Definition: Noun phrase and indeterminate noun phrase in J}

This consists of two parts.

Part 1. (Indeterminate terms)

1. If N is a noun, then “(dono N)” is an indeterminate term.
2. Nothing is an indeterminate term unless it is so by the above.

Part 2.

3. A noun and an indeterminate term are noun phrases. An indeterminate term is an indeterminate noun phrase.
4. If \(\alpha\) is a noun phrase and \(\nu\) is a noun or an indeterminate term, then “((\(\alpha\) no) \(\nu\))” is a noun phrase. If \(\alpha\) or \(\nu\) is an indeterminate noun phrase, then so is “((\(\alpha\) no) \(\nu\))”.
5. If N is a noun and \(\nu\) is a noun or an indeterminate term, then “(((dono N) ka) no) \(\nu\))” is a noun phrase. If \(\nu\) is an indeterminate noun phrase, then so is “(((dono N) ka) no) \(\nu\))”.
6. If \(\nu\) is a noun or an indeterminate term and IV is a one-place (intransitive) verb\(^2\), then “(IV \(\nu\))” is a noun phrase. If \(\nu\) is an indeterminate noun phrase, then so is “(IV \(\nu\))”.

\(^2\) Strictly speaking, a verb has to be in an attributive form (rentai-kei). The same applies to other kinds of verbs, too.
7. If $\alpha$ is a noun phrase, $\nu$ is a noun or an indeterminate term, and TV is a two-place (transitive) verb, then “(((\alpha \text{ ga} TV) \nu))” and “(((\alpha \text{ o} TV) \nu))” are noun phrases. If $\alpha$ or $\nu$ is an indeterminate noun phrase, then so are “(((\alpha \text{ ga} TV) \nu))” and “(((\alpha \text{ o} TV) \nu))”.

8. If $\alpha$ is an indeterminate noun phrase, $\nu$ is a noun or an indeterminate term, and TV is a two-place verb, then “(((\alpha \text{ mo ga} TV) \nu))”, “(((\alpha \text{ o mo} TV) \nu))”, and “(((\alpha \text{ mo} TV) \nu))” are noun phrases. If $\nu$ is an indeterminate noun phrase, then so are “(((\alpha \text{ mo ga} TV) \nu))”, “(((\alpha \text{ o mo} TV) \nu))”, and “(((\alpha \text{ mo} TV) \nu))”.

9. If N is a noun, $\nu$ is a noun or an indeterminate term, and TV is a two-place verb, then “(((\text{dono} N) \text{ ka ga} TV) \nu))” and “(((\text{dono} N) \text{ ka o} TV) \nu))” are noun phrases. If $\nu$ is an indeterminate noun phrase, then so are “(((\text{dono} N) \text{ ka ga} TV) \nu))” and “(((\text{dono} N) \text{ ka o} TV) \nu))”.

10. Nothing is a noun phrase or an indeterminate noun phrase unless it is so by the above.

To see how this definition works, let us see some examples. First, any of the following expressions are indeterminate noun phrases by clauses 1, 3 and 4.

- (\text{dono kagi}) which key
- (((\text{heya no}) (\text{dono kagi})) which key of room(s)
- (((\text{dono hey}a \text{ no}) \text{ kagi}i) key(s) of which room
- (((\text{dono hey}a \text{ no}) (\text{dono kagi})) which key of which room

Here is another one in the same line.

- (((\text{tatemono no}) (\text{dono hey}a)) \text{ no} (\text{dono kagi}))
  building GEN which room GEN which key
  (which key of which room of building(s))

Secondly, here are some examples of indeterminate noun phrases that are shown to be so by clauses 1, 6 and 7. I give them without any parsing by parentheses for simplicity’s sake.

(a) \text{waratta} \text{ dono kodomo}
  laughed which child
  (which child who laughed)

(b) \text{dono kodomo} ga \text{ yonda hon}
  which child NOM read book
(book(s) that which child read)

c) kodomo o oshieta dono sensei
   child ACC taught which teacher
   (which teacher that taught child(ren))

d) dono kodomo o oshieta sensei ga yonda dono hon
   which child ACC taught teacher NOM read which book
   (which book that teacher(s) that taught which child)

One thing we should be clear about this definition is that, according to it, an indeterminate noun phrase is not the same as a noun phrase which contains an occurrence of an indeterminate word like “dono”. For example,

e) dono kodomo o mo oshieta sensei
   which child ACC ∀ taught teacher
   (teacher(s) who taught every child)

is a noun phrase by clauses 1 and 8, but it is not an indeterminate noun phrase, although it contains an occurrence of “dono”. In contrast,

f) dono kodomo o mo oshieta dono sensei
   which child ACC ∀ taught which teacher
   (which teacher who taught every child)

is an indeterminate noun phrase by the same clauses 1 and 8 of the definition. A criterion to judge whether a given noun phrase is an indeterminate noun or not is to see whether it can be combined with “mo” to form a universal sentence. (f) can be combined with “mo” and form a universal statement as the following sentence shows.

(70) dono kodomo o mo oshieta dono sensei mo kita
   which child ACC ∀ taught which teacher ∀ came
   (Every teacher who taught every child also came.)

It is true that we can combine (e) with “mo”, but the resulting sentence is not a universal statement. In the following sentence, “mo” is not a universal quantifier particle, but a particle which is sometimes called “focus particle (toritate jyoshi)” and means “also” or “too”.

(71) dono kodomo o mo oshieta sensei mo kita
   which child ACC ∀ taught teacher also came
   (A teacher/Teachers who taught every child also came.)

While a universal quantifier particle “mo” may be applied to any indeterminate noun phrase that is characterized above, an existential quantifier particle
“ka” can be applied only to a simple indeterminate noun phrase. This fact is reflected in clauses 5 and 7. “Ka” can come only right after an indeterminate term “(dono N)”. The present definition can be extended to include other indeterminate words like “dare” (who), “dore” (which) and “nani” (what) among indeterminate terms. In that case, clauses 1, 5 and 9 should be generalized to cover them in the following way.

1’. “dare”, “dore” and “nani” are indeterminate terms. If N is a noun, then “dono N” is an indeterminate term.

5’. If μ is an indeterminate term and ν is a noun or an indeterminate term, then “(((μ ka) no) ν)” is a noun phrase. If ν is an indeterminate noun phrase, then so is “(((μ ka) no) ν)”.

9’. If μ is an indeterminate term, ν is a noun or an indeterminate term, and TV is a two-place verb, then “(((μ ka) ga) TV) ν)” and “(((μ ka) o) TV) ν)” are noun phrases. If ν is an indeterminate noun phrase, then so are “(((μ ka) ga) TV) ν)” and “(((μ ka) o) TV) ν)”.

Under this new definition, of the following two noun phrases, only the second one will be an indeterminate noun phrase.

(g) dare ka o oshieta sensei
who ∃ ACC taught teacher(s)

3 According to the present definition, an expression such as

(i) dono kodomo ka no hon
which child ∃ GEN book(s)

(some child’s book(s))

is a noun phrase (clause 5), while an expression that has “mo” instead of “ka”

(ii) ?dono kodomo mo no hon
which child ∀ GEN book(s)

(Supposedly: every child’s book(s))

is not a noun phrase. It is because I am not sure whether an expression like (ii) is grammatically all right. At least, it seems certain that, just as is the case with “ka”, “mo” cannot be applied to a complex noun phrase which itself contains an occurrence of “no” to form a larger noun phrase with “no”.

(iii) *dono kodomo no sensei mo no hon
which child GEN teacher(s) ∀ GEN book(s)

(Supposedly: a book/books of every child’s teacher(s))

Of course, there is nothing wrong in the combination of an indeterminate noun phrase which contains an occurrence of “no” and a universal quantifier particle in other contexts, as the following example shows.

(iv) dono kodomo no sensei mo kita
which child GEN teacher(s) ∀ came

(For every child, its teacher(s) came.)
It is not difficult to extend the present definition to include verbs that take more than two arguments, if we don’t mind the work involved. In such an extension, expressions like the following will be recognized as indeterminate noun phrases.

(i) dono sensei ga dono hon o ageta seito
    which teacher NOM which book ACC gave pupil
    (a pupil/pupils whom which teacher gave which book)

(j) dono seito ni dono hon o ageta dono sensei
    which pupil DAT which book ACC gave which teacher
    (which teacher who gave which book to which pupil)

We find in the literature some examples in which an indeterminate phrase occurs in a complement of a propositional attitude verb such as this.

(k) Dono e ga yoi to omotta hito
    which picture NOM good QUOT thought person
    mo kita
    \( \forall \) came
    (Any person who thought good of some picture came.)

It is a remarkable fact that “mo” can “quantify into” such a hyper-intensional context. I will discuss this context in chapter 8.

### 4.3 “Dono” as a singularizing predicate

As we have now a characterization of the expressions that are involved in indeterminate phrase quantification with “dono”, let us start with the first question, namely, what makes “dono” quantification a singular quantification.

In a sentence like (61), in which a simple indeterminate phrase of the form “dono + N” immediately precedes “mo”, “dono + N + mo” means a singular quantification “each N” or “every N”. But, when we look at the sentences (64) and (65), we notice that, although the noun phrases that precede “mo” begin with “dono”, they themselves may not be singular. In (64), each child might have written a number of letters, and in (65), for some children, both of their parents might have come. Hence, it looks very likely that singularity comes from “dono” and that “mo” is not by itself a singular quantifier.
This is supported by the fact that “dono + N” means always “each N” even when it occurs as part of a larger noun phrase that precedes “mo”. (64) talks about each child’s parent or parents, and (65) talks about a letter or letters each child wrote. We might use “kodomo” to refer to some children collectively, but by using “dono kodomo” we talk about them in a distributive way, namely, saying that something is true of each of them.

A natural thought here is that “dono” in “dono + N” cuts down the extension of the noun N to individuals only. As is the case with most Japanese nouns, “kodomo” has as its extension different groups of children as well as individual children. “Dono kodomo” has a narrower extension that consists of only individual children. A simple way to express this is to adopt an axiom like the following.

**Axiom of “dono + N”**  (the first version)

Let N be a noun. Then,


Remember that “IX” means that X is an individual, and that I is a logical predicate in plural logic. How this axiom works can be seen from this example.

Val(X, “kodomo”) ⇔ X is/are a child/children.

Val(X, “dono kodomo”) ⇔ X is an individual ∧ Val(X, “kodomo”).

This axiom seems to work fine also in more complex cases. Take a complex indeterminate noun phrase “dono kodomo no oya” (each child’s parent(s)), which occurs in (64). “Oya” (parent) is what I call a relational noun which takes an argument, and its semantics is given by the following axiom.

**Axiom of a relational noun**.

4 The use of an expression “group” here should not be construed as implying the existence of special kind of entities groups. A group of people is just a number of peoples.

5 [Iida 2013].

6 There must also be an axiom for a similar construction with a non-relational noun instead of a relational one, like “dono heya no kagi” (key(s) of which room). Such a construction should be distinguished from an adjectival construction which we encountered in §3.1.1. Here I suppose that “(NP no) N” is a construction which roughly corresponds to an English possessive construction “NP’s N”. Then, the axiom in question can be given in the following way.

**Axiom of possessive construction with non-relational noun**

Let NP be a noun phrase and N a non-relational noun, then

Val(X, “(NP no) N”) ⇔ ∃Y[Val(Y, NP) ∧ [Val(X, N) ∧ RC(X, Y)]]],

where RC is a certain relation determined by a context C.
Let NP be a noun phrase and RN a relational noun, then

\[ \text{Val}(X, "(NP \text{ no} \ RN") \Leftrightarrow \exists Y (\text{Val}(Y, NP) \land \text{Val}((X, Y), RN)) \]  \]  

With this axiom and the previous one, we can derive what semantic values “dono kodomo no oya” has.

1. \( \text{Val}(X, "((\text{dono kodomo}) \text{ no} \ oya") \)
2. \( \exists Y (\text{Val}(Y, "(\text{dono kodomo})") \land \text{Val}((X, Y), "\text{oya")}) \)
3. \( \exists Y ([IY \land \text{Val}(Y, "\text{kodomo")}) \land \text{Val}((X, Y), "\text{oya")}) \)
4. \( \exists Y [Y \text{ is an individual child} \land X \text{ is/are } Y\text{'s parent(s)}] \)

If we remember a convention in plural logic that lower-case letters like \( x, y, z \) are variables that range over only individuals, then the last line of the above can be rewritten thus.

\[ \exists y [y \text{ is a child} \land X \text{ is/are } y\text{'s parent(s)}] \)

Thus, if singularization is the work of “dono”, then we may assume that “mo” is simply a (plural) universal quantifier that has an indeterminate noun phrase as its restriction. Its semantics is straightforward. Syntactically, however, there are some complications relating to the co-occurrence of “mo” and a case particle. “Mo” sometimes occurs with a case particle as in (5). Although “mo” occurs without a case particle in (61), it might do so as well. If \( \alpha \) is an indeterminate noun phrase and \( \phi \) a predicate, a general pattern is this, provided that we are concerned with only verbs with one or two arguments and predicates constructed from them.

\[ (\alpha \text{ mo } [\text{ga}]) \phi \]
\[ (\alpha \text{ [o] mo }) \phi \]

A relation \( R_{\varphi} \) between the things denoted by NP and N can be very diverse. Take a noun phrase

\[
\text{sensei} \quad \text{no} \quad \text{hon.} \\
\text{teacher(s)} \quad \text{GEN} \quad \text{book(s)}
\]

As its English counterpart “a teacher’s book”, it may mean any of the following: a book written by a teacher, a book possessed by a teacher, a book about a teacher.

\[ 7 \] A few words should be said about the use of an ordered pair \((X, Y)\) in this axiom. As we are working within a plural logic framework, an ordered pair should not be identified with a certain set as is customary done. We regarded an ordered pair as one of the primitives of our metalanguage. There is no reason not to do so.
Using this convention, we may give a semantic axiom for “mo” in the following way. As we are going to revise it in later stages of our discussion, it is termed as “the first version”.

**Axiom of a universal quantifier “mo”** (the first version)

Let $\alpha$ be an indeterminate noun phrase.

(i) If $\phi$ is a one-place predicate, then “($\alpha$ mo $\phi$)” is true if and only if

$$\forall X [\text{Val}(X, \alpha) \rightarrow \text{Val}(X, \phi)].$$

(ii) If $\phi$ is a two-place predicate, then $\text{Val}(X, (\alpha \ [\text{ga}] \ mo) \phi)$ if and only if

$$\forall Y [\text{Val}(Y, \alpha) \rightarrow \text{Val}((Y, X), \phi)],$$

and $\text{Val}(X, (\alpha \ [\text{o}] \ mo) \phi)$ if and only if

$$\forall Y [\text{Val}(Y, \alpha) \rightarrow \text{Val}((X, Y), \phi)].$$

In case (i), “($\alpha$ mo $\phi$)” might be “($\alpha$ mo [ga] $\phi$)” as well as “($\alpha$ [o] mo $\phi$)”. We are going to call “$\alpha$” as “the restriction of ‘mo’” and $\phi$ as “the scope of ‘mo’”.

Our axiom of “mo” is not general enough to deal with predicates of more than two places, but I suppose it is not difficult to extend it to them.

To see how this axiom works, take (61), which I repeat here.

(61) Dono kodomo mo waratta.

Every child laughed.

By our axiom of “mo”, we get

(72) $\forall X [\text{Val}(X, (“{\text{dono kodomo}}”)) \rightarrow \text{Val}(X, “\text{waratta}”).]

Then, by axiom of “dono + N”,

(73) $\forall X [\text{Val}(X, “{\text{kodomo}}”)] \rightarrow \text{Val}(X, “\text{waratta}”).]

From this, with lexical axioms for “kodomo” and “waratta” and changing a plural variable into a singular one, we get

(74) $\forall x [x$ is a child $\rightarrow x$ laughed].
which is as it should be.

If we look at the definition of an indeterminate noun phrase in the previous section, we notice that there are three patterns in which “mo” associates with “dono”. First, “mo” associates with a simple indeterminate noun phrase of the form “dono + N” which immediately precedes it. Secondly, “mo” associates with “dono” which appears in a noun clause that modifies another noun phrase connected by “no”. Thirdly, “mo” associates “dono” which appears in a predicate that modifies a noun phrase.

We have seen how the present semantics works for the first case. Let us turn to the second case, of which (64) is an example.

(64) Dono kodomo no oya mo kita.
    (Any child’s parent(s) came.)

By our axiom of “mo”, we get the following as the truth condition of (64).

(75) ∀X[Val(X, “(((dono kodomo) no) oya)”) → Val(X, “kita”)]

Given the result about the semantic values of “dono kodomo no oya” and a lexical axiom for “kita” (came), we get

(76) ∀X[∃y[y is a child ∧ X is/are y’s parent(s)] → X came]

This can be paraphrased as

(77) For all X, if X is/are a parent/parents of some child, then X came.

(76) has the form

∀X[∃yα(X, y) → ϕ(X)],

which is equivalent to

∀X∀y[α(X, y) → ϕ(X)],

This equivalence holds only if “ϕ(X)” does not contain y free, and this condition is satisfied in the present case. Hence, (77) can be again paraphrased as

(78) For all X, and each y, if X is/are a parent/parents of y, then X came.

This seems to be the right result.

The remaining case is the one in which “mo” associates with “dono” that appears in a noun modifying predicate. Let us take (65) as an example.
(65) Dono kodomo ga kaita tegami mo todoita.

(Any letter(s) that any child wrote arrived.)

By the axiom of “mo”, (65) is true if and only if

(79) ∀X[Val(X, “((((dono kodomo) ga) kaita) tegami)”) → Val(X, “todoita”)].

Let us see how the antecedent of the conditional within the scope of “∀X” is unpacked.

1. Val(X, “(((dono kodomo) ga) kaita) tegami)”
2. Val(X, “((dono kodomo) ga) kaita”) ∧ Val(X, “tegami”)
   ∧ Val(X, “tegami”)
   ∧ Val(X, “tegami”)]
5. ∃y[y is a child ∧ y wrote X] ∧ X is/are a letter/letters

As we have not yet introduced all the semantic axioms that are needed to justify this derivation, I briefly indicate them.

The step from line 1 to line 2 is justified by an axiom about a complex noun phrase consisting of a noun modifying predicate and a modified noun. What we need here is an application of this axiom to a case with a one-place predicate ϕ and a simple noun phrase ν:

Axiom of noun modifying predicate

Val(X, “(ϕ ν”) ⇔ Val(X, ϕ) ∧ Val(X, ν).

Line 3 is obtained from line 2 by applying an axiom similar to the predication axiom for monotone increasing case (§3.1.1) to the modifying clause “dono kodomo ga kaita”.

I suppose lines 4 and 5 do not present anything new; they are justified by an axiom of “dono + N” and our convention for lower-case variables.

Thus, (79) is equivalent to the following.

(80) ∀X[∃y[y is a child ∧ y wrote X] ∧ X is/are a letter/letters]
   → X arrived],

which is, in turn, equivalent to

(81) ∀X∀y[[y is a child ∧ y wrote X] ∧ X is/are a letter/letters]
   → X arrived].

It seems that we are led to the right result again.
4.4 Why “ka” does not have a long-distance association

Moreover, it seems that we have a natural explanation why unlike “mo” “ka” does not have a long-distance association.

Now suppose perhaps counterfactually that “ka” were admitted to a position away from an indeterminate phrase and (68) were grammatically all right.

(68) Dono kodomo ga kaita tegami ka ga todoita.
    child(ren) NOM wrote letter(s) ∃ NOM arrived
    (Supposedly: Some letter(s) which some child wrote arrived.)

Let us suppose that “ka” is an existential quantifier and its semantics is given by an axiom similar to the axiom of “mo”. Unlike “mo”, “ka” should be accompanied by a case particle, and “ka” always precedes a case particle.

**Axiom of an existential quantifier “ka”**

Let α be an indeterminate noun phrase.

(i) If ϕ is a one-place predicate, then
   “((α ka) ga) ϕ” is true ⇔ ∃X[Val(X, α) ∧ Val(X, ϕ)].
   “((α ka) o) ϕ” is true ⇔ ∃X[Val(X, α) ∧ Val(X, ϕ)].

(ii) If ϕ is a two-place predicate, then
    Val(X, “((α ka) ga) ϕ’’) ⇔ ∃Y[Val(Y, α) ∧ Val(Y, X, ϕ)].
    Val(X, “((α ka) o) ϕ’’) ⇔ ∃Y[Val(Y, α) ∧ Val(X, Y, ϕ)].

Let us derive the truth condition of (68) using this axiom and our account of “dono” as a singularizing predicate. It is easily shown that

Val(X, “(((dono kodomo) ga) kaita) tegami)"

is equivalent to

∃y[y is a child ∧ y wrote X] ∧ X is/are letters.

Hence, by the axiom of “ka”, we conclude that (72) is true if and only if

(*) ∃X[∃y[y is a child ∧ y wrote X] ∧ X is/are letters] ∧ X arrived].

We should compare this result with the truth condition of the corrected version of (68), in which “ka” appears right after “dono kodomo”.

52
Dono kodomo ka ga kaita tegami ga todoita.
(Some letter(s) which some child wrote arrived.)

As (68cor) has the form “indefinite NP + predicate”, it is true if and only if the subject NP and the predicate have some semantic values in common. So, let us first see what the semantic values of the indefinite noun phrase are. 1 is equivalent to the conjunction of the following two.

1. Val(X, “(((dono kodomo) ka) ga) kaita) tegami”)

Applying case (ii) of Axiom of “ka” to 2, we see that it is equivalent to


By Axiom of “dono + N”, 4 is equivalent to

5. ∃Y[IY ∧ Val(Y, “kodomo”) ∧ Val(⟨Y, X⟩, “kaita”)]

Changing a plural variable to a singular one and by lexical axioms for “kodomo” and “kaita”, we have

6. ∃y[y is a child ∧ y wrote X]

With another conjunct 3, we know that 1 is equivalent to

7. ∃y[y is a child ∧ y wrote X] ∧ X is/are a letter/letters.

Thus, the truth condition of (68cor) is this.

(**) ∃X[∃y[y is a child ∧ y wrote X] ∧ X is/are letters] ∧ X arrived]

There is no difference between the truth condition of (68cor) and that of “ungrammatical” (68). This result can be easily foreseen. For, what “ka” contributes to the sentence in which it occurs is an existentially quantified clause and, if the whole sentence itself is existentially quantified like (68cor), the scope of “ka” does not make any difference. Whether “ka” has a narrow scope as in (68cor) or a wide scope as in (64), the results will be the same.

The matter is totally different with universal quantifier “mo”. “Mo” marks the antecedent, and if it were placed in a different position of the sentence,
then the antecedent becomes different and that makes the difference to its truth condition.

If I may be allowed to speculate a little, then the reason why “ka” does not have a long distance association might be something like the following. If “ka” could have a wide scope, we would have to determine what its scope is whenever we encounter “ka”. But, if it does not make any difference to the sentence in which “ka” appears whether its scope is wide or not, then the trouble we should take for determining the exact scope of “ka” is not worth it. Thus, there is no point in letting “ka” to have a wide scope.

While I am at it, here is another speculation. Although “mo” is not a singular quantifier, if its restriction is a simple indeterminate noun phrase “dono + N”, it seems to work as a singular universal quantifier. The fact that “mo” is not intrinsically singular can be seen from sentences like (64) and (65) in which “mo” is associated with “dono + N” embedded in a complex noun phrase. We might say the same thing with “ka”, namely, that “ka” is not intrinsically singular quantifier but a plural existential quantifier. But, as “ka” occurs only with a simple indeterminate noun phrase and never with a complex one, “ka” can occur only in singular existential quantification. Our “ungrammatical” (68) seems to show that, if “ka” could have a wide scope, then it could work as a plural quantifier.
Chapter 5

“Dono + N” as a sorted variable

Then, do we have a satisfactory account of “dono” quantification? Unfortunately, the answer is no. There are at least two problems with the present account.

One of them is that the derived conditions for (64) and (65) are too strong. Take (81), which is supposed to be the truth condition of (65). If we look at it carefully, we notice that it implies that all the letters any child wrote arrived. But, (65) does not make such a strong statement. It is not necessary that every letter each child wrote arrived; (65) will be true if for every child who wrote letters some of them arrived\(^1\). The same thing can be said with (64) and (75). For (64) to be true it is not necessary that both parents of each child came. If every child has at least one parent who came, then (64) will be true. But, (75) implies that for every child both of its parents came. Again, the derived condition is stronger than the original.

Does this mean that our supposition that “mo” is a plural universal quantifier is wrong? I don’t think so. I am going to explain why I don’t think so and how we can get the right truth condition for a sentence like (64) and (65) in the next chapter.

The other problem is a much more basic one, however. It is that the present account does not meet the second of our desiderata for a satisfactory account, namely, to explain why an indeterminate phrase should always appear with “mo” or “ka”.

If the only difference between “kodomo” and “dono kodomo” is that the latter has only individual children as its extension, then it should have the same

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\(^1\) Does (65) imply that every child wrote a letter or letters? In coming sections, we will be concerned with this question as a part of the problem of presupposition involved in indeterminate phrase quantification.
meaning as “choudo hitori no kodomo (exactly one child)” which also has only individual children as its extension. But, this noun phrase never makes a universally quantified sentence with “mo”. An indeterminate phrase like “dono kodomo” demands “mo” or “ka”, unless it occurs in an interrogative sentence. Hence, the function of “dono” cannot simply consist in cutting down the extension of the associated noun to individuals only.

5.1 The concept of assignment and semantics of a variable

Let us remember how quantifiers work in the language of predicate logic, the standard language of modern logic. A quantifier does not attach to a sentence, but to an expression which is called an open sentence. An open sentence is not a sentence in the full-fledged sense because it contains variables at the places where usually names are. Naturally, if you substitute an appropriate name to each variable, then an open sentence becomes a genuine one. But, there is another way of turning an open sentence into a genuine one. It is to quantify it, that is, to put quantifiers with suitable variables in front of an open sentence. In this process, variables are said to be bound by quantifiers. Variables which are not bound by any quantifiers are called “free”.

Although our indeterminate noun phrases are noun phrases and not sentences, an obvious suggestion here is that they are like open sentences (open noun phrases?) and that they contain variable-like expressions. What can be these variable-like expressions? They can be nothing but indeterminate words. Quantifier particles “mo” and “ka” which come after an indeterminate noun phrase “bind” an indeterminate word (or indeterminate words) that appears in the noun phrase and turns it to something which is no longer an indeterminate one.

How should we implement such an idea? If an indeterminate word is like a variable, then we had better to take a look at how the semantics of a variable is given in the standard logical language.

One of the main ingredients of the semantics of a variable is the concept of assignment. A variable is associated with a certain domain; a first-order variable is associated with a domain of individuals. An assignment is something which assigns an object or a number of objects in the domain to a variable. If a variable is singular, it is assigned an object; if it is plural, it is assigned a number of objects. If two assignments assign the different objects or different groups of objects to the same variable, then they are different assignments. Usually, an assignment is a simultaneous assignment for a number of different variables. An assignment can be represented by a list of pairs consisting of a variable and an object (or a number of objects) assigned to it. In each assignment, the same variable cannot be paired with different objects (or different groups of objects),
but the same object (or the same group of objects) can be paired with different
variables.

Suppose we have three singular variables “a₁”, “a₂” and “a₃”, which have
the common domain consisting of three persons, Hanako, Taro, and Jiro. Then,
there are $3^3$ different assignments defined on these three variables, some of which
are here.

\[
\begin{align*}
\sigma_1 : & \langle “a_1”, \text{Hanako} \rangle, \langle “a_2”, \text{Taro} \rangle, \langle “a_3”, \text{Jiro} \rangle \\
\sigma_2 : & \langle “a_1”, \text{Hanako} \rangle, \langle “a_2”, \text{Hanako} \rangle, \langle “a_3”, \text{Jiro} \rangle \\
\sigma_3 : & \langle “a_1”, \text{Taro} \rangle, \langle “a_2”, \text{Hanako} \rangle, \langle “a_3”, \text{Taro} \rangle
\end{align*}
\]

Semantic value relation for a variable should be relativized to an assignment.
Suppose that $\sigma$ is an assignment and $v$ is a variable which is assigned an object
by $\sigma$. As an assignment pairs the unique object with each variable $v$, we may
denote the object which $\sigma$ assigns to $v$ by “$\sigma(v)$”. Then, the following holds in
general.

\[\text{Val}(X, v, \sigma) \iff X \equiv \sigma(v)\]
As we are going to consider only assignments to singular variables here, we can
switch to a lower-case metavariable “x”. Hence, for any object $x$,

\[\text{Val}(x, v, \sigma) \iff x = \sigma(v)\]

For example, if assignments $\sigma_1$, $\sigma_2$, and $\sigma_3$ are given as above, then

\[
\begin{align*}
\text{Val}(x, “a_1”, \sigma_1) \iff & \quad x = \sigma_1(“a_1”) \\
\iff & \quad x = \text{Hanako} \\
\text{Val}(x, “a_2”, \sigma_3) \iff & \quad x = \sigma_3(“a_2”) \\
\iff & \quad x = \text{Hanako} \\
\text{Val}(x, “a_3”, \sigma_2) \iff & \quad x = \sigma_2(“a_3”) \\
\iff & \quad x = \text{Jiro}
\end{align*}
\]

Let us see whether a similar account can be given to “dono + N”. Take “dono
kodomo no oya” which is found in (64). This is an indeterminate noun phrase
which contains another indeterminate noun phrase “dono kodomo”. We might
suppose that “dono + N” is a singular variable whose domain consists of those
individual things that can be semantic values of a noun N. For example, “dono
ekodomo” is a singuar variable, and its domain consists of individual children.

This means that “dono” is an expression which turns a noun into a xsorted
singular variable. A sorted variable is a variable whose domain is restricted to a
certain sort of objects. For example, it frequently happens in mathematics that
“n” and “m” are used as variables for natural numbers, and “x” and “y” for
real numbers; they are sorted variables. A sorted variable is used when we wish
to work with a number of different domains at the same time instead of one
universal domain. Different styles of variables reflect differet domains. Hence,
we can tell the domain of a variable from its style. As a matter of fact, we have
been using sorted variables in our metalanguage; namely, we are using a capital letter like “X” as a plural variable and a lower-case letter like “x” as a singular variable.

Our claim is that “dono + N” is a sorted variable whose domain is indicated by a noun N. Moreover, this domain is not the same as the extension of N; it consists of only individuals among the semantic values of N. Hence, “dono + N” is a sorted singular variable. This is the reason why “dono + N” quantification is singular.

How many different assignments are there for “dono kodomo”? If we consider only assignments which assign a certain child to this expression and do nothing else, then there are different assignments as many as individual children. Suppose that there are only three children, Hanako, Taro and Jiro. Then, there are three different assignments for “dono kodomo”, namely,

\[ \sigma_1 : \text{“}(\text{dono kodomo})\text{”, Hanako} \]
\[ \sigma_2 : \text{“}(\text{dono kodomo})\text{”, Taro} \]
\[ \sigma_3 : \text{“}(\text{dono kodomo})\text{”, Jiro} \]

Now, “dono kodomo” has its semantic value only relative to an assignment. For example,

\[ \text{Val}(x, \text{“}(\text{dono kodomo})\text{”, } \sigma_1) \iff x = \sigma_1(\text{“}(\text{dono kodomo})\text{”}) \iff x = \text{Hanako} \]

Given this, we can derive the following, provided that our axiom of a relational noun in the previous section is relativized to an assignment\(^2\).

1. \( \text{Val}(X, \text{“}(\text{dono kodomo}) \text{ no) oya“}, \sigma_1) \)
2. \( \exists Y[\text{Val}(Y, \text{“}(\text{dono kodomo})\text{”, } \sigma_1) \land \text{Val}(\langle X, Y \rangle, \text{“oya“}, \sigma_1)] \)
3. \( \exists Y[Y \equiv \sigma_1(\text{“}(\text{dono kodomo})\text{”}) \land \text{Val}(\langle X, Y \rangle, \text{“oya“})] \)
4. \( \exists Y[Y \equiv \sigma_1(\text{“}(\text{dono kodomo})\text{”}) \land X \text{ is/are a parent/parents of } Y] \)
5. \( X \text{ is/are a parent/parents of } \sigma_1(\text{“}(\text{dono kodomo})\text{”}) \)
6. \( X \text{ is/are Hanako’s parent(s).} \)

\(^2\) We also need another axiom which connects a relativized version with unrelativized version, namely,

For any assignment \( \sigma \), \( \text{Val}(\langle X, Y \rangle, \text{“oya“}, \sigma) \iff \text{Val}(\langle X, Y \rangle, \text{“oya“}) \),

which is a particular application of a general axiom

If \( \alpha \) is an expression which does not have a free occurrence of an indeterminate noun phrase in the sense that will be explained later, then

\[ \text{for any assignment } \sigma, \forall X[\text{Val}(X, \alpha, \sigma) \iff \text{Val}(X, \alpha)]. \]
As an assignment may be a simultaneous one to different variables, we may have to consider a simultaneous assignment to different indeterminate terms of the form “dono + N”. Sometimes we have to consider even a simultaneous assignment to different occurrences of the same “dono + N” phrase as in the following example.

(82) Dono kodomo ga dono kodomo ni dashita tegami mo todoita.
    which child NOM which child DAT sent letter(s) ∀ arrived
    (Letters each child sent to each child arrived.)

Moreover, it is not true that every occurrences of a “dono + N” phrase in a particular indeterminate noun phrase should be assigned an object by an assignment for that phrase. Consider (70), which I repeat here.

(70) Dono kodomo o mo oshieta dono sensei mo kita
    which child ACC ∀ taught which teacher came
    (Every teacher who taught every child came.)

In this sentence, an indeterminate noun phrase which functions as the restriction of the second occurrence of “mo” is this.

(e) dono kodomo o mo oshieta dono sensei
    which child ACC ∀ taught which teacher

There are two occurrences of indeterminate terms in this. However, as far as we are considering them in the context of (e), the first one, namely, “dono kodomo” is already bound by “mo” which comes immediately after it, and is no longer a variable-like expression that needs to be assigned a value. (e) itself is an indeterminate noun phrase because of the occurrence of “dono sensei” at the end.

This example teaches us two things. First, it is necessary to characterize when an occurrence of an indeterminate term is “free” and should be given a value by an assignment. Only free occurrences of an indeterminate term are to be assigned objects by an assignment.

Secondly, it makes sense only relative to a linguistic context whether an occurrence of an indeterminate term is free or not. Although the occurrence of “dono kodomo” in (e) cannot be assigned a value by any assignment because it is already bound in (e), if it is considered by itself, or considered relative to itself, then it should be given a value by an assignment. In fact, in the course of giving a semantic account of (e) in terms of assignments relative to (e), we should also account a quantification inside (e) and consider assignments relative
to “dono kodomo” itself and in that context the occurrence of “dono kodomo” should be regarded free.

Fortunately, it is not difficult to characterize when an occurrence of an indeterminate term is free. Here is a definition of free occurrence of an indeterminate term relative to an indeterminate noun phrase in which it occurs.

**Definition: a free occurrence of an indeterminate term**

An occurrence $D$ of an indeterminate term in an indeterminate noun phrase $\alpha$ is free if and only if there is no occurrence of “mo” or “ka” after $D$ in $\alpha$.

The occurrence of “dono kodomo” in (e) is not free because “mo” occurs right after it, while that of “dono sensei” in (e) is free because there is no occurrence of “mo” or “ka” after that. In the above definition, $\alpha$ might be $D$ itself. Thus, the occurrence of “dono kodomo” in “dono kodomo” itself is free because “dono kodomo” does not contain any occurrence of “mo” or “ka”.

In general, if “mo” occurs within a noun phrase, it binds all the preceding occurrences of indeterminate terms if they are not already bound by any preceding “mo” or “ka”. In contrast, “ka” binds only an immediately preceding indeterminate term. But, if there were an indeterminate term before “ka” that is neither right before it nor bound by any preceding “mo” or “ka”, the expression would be grammatically suspect as we observed before with (67)–(69). Hence, we can safely say that anything which comes before “ka” cannot be a free occurrence of an indeterminate term$^3$.

Now we are ready to define the concept of assignment. It is enough to consider only assignments to indeterminate terms that are found within an indeterminate noun phrase $\alpha$ which can be a restriction of a universal quantifier particle “mo” for the reason that will be explained later. So, we define an assignment relative to such $\alpha$ and call it “an assignment for $\alpha$”.

**Definition: an assignment for an indeterminate noun phrase**

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$^3$ There is also an interrogative “ka”. But the above definition is valid even when “ka” is an interrogative particle. For example, in

(i) Dono kodomo ga kita ka shitte-iru
dono sensei mo kaetta.

(child NOM came ? know
teacher ∀ went home)

(Every teacher who knows which child came went home.)

“dono kodomo” is bound by an interrogative “ka”, but “dono sensei” was free before it was bound by “mo”. There will be a further discussion relating to this in §7.2.

$^4$ Note that an indeterminate term by itself can be a restriction to “mo”. In such a case, $\alpha$ is the same as the indeterminate term in question.
Let $\alpha$ be an indeterminate noun phrase and $D_1, D_2, \ldots, D_n$ be the first, second, \ldots, and $n$-th occurrences of indeterminate terms of the form \textit{“(dono $N_k$)“} ($1 \leq k \leq n$) which are free in $\alpha$. Then, an assignment $\sigma$ for $\alpha$ is a list of $n$ pairs

$$\langle D_1, x_1 \rangle, \langle D_2, x_2 \rangle, \ldots, \langle D_n, x_n \rangle.$$ 

such that, for each $k(1 \leq k \leq n)$,

if $N_k$ is a non-relational noun, then $\operatorname{Val}(x_k, N_k)$.

if $N_k$ is a relational noun, then $\exists Y \operatorname{Val}(\langle x_k, Y \rangle, N_k)$.

An assignment for $\alpha$ is written as “$\sigma^\alpha$”. As each $\sigma^\alpha$ pairs the unique object $x_k$ with each $D_k$, by $\sigma^\alpha(N_k, k)$

we denote an individual which is paired by $\sigma^\alpha$ with the $k$-th occurrence in $\alpha$ of an indeterminate term of the form \textit{“dono $N_k$“}.

For any two $D_k$’s, which object is paired with them in an assignment is independent from each other. Hence, for any sequence of $n$ objects

$$\langle x_1, x_2, \ldots, x_n \rangle$$

such that, for each $k(1 \leq k \leq n)$,

$\operatorname{Val}(x_k, N_k)$, if $N_k$ is a non-relational noun,

$\exists Y \operatorname{Val}(\langle x_k, Y \rangle, N_k)$, if $N_k$ is a relational noun,

there is an assignment $\sigma^\alpha$ such that, for each $k(1 \leq k \leq n)$,

$$x_k = \sigma^\alpha(N_k, k)$$

in other words, $\sigma^\alpha$ is an assignment which assigns $x_k$ for $D_k$ for each $k$ such that $1 \leq k \leq n$.

We note that the following holds from the definition of an assignment for an indeterminate noun phrase.

**Proposition**

Let $\sigma^\alpha$ be an assignment for $\alpha$. For any $k$ such that $1 \leq k \leq n$,

(C1) if $N_k$ is a non-relational noun, then

$$\exists x [\operatorname{Val}(x, N_k) \land \sigma^\alpha(N_k, k) = x]$$

or,

$$\operatorname{Val}(\sigma^\alpha(N_k, k), N_k)$$
(C2) if $N_k$ is a relational noun, then

$$\exists x [\exists Y \text{Val}(\langle x, Y \rangle, N_k) \land \sigma^\alpha(N_k, k) = x]$$

or,

$$\exists Y \text{Val}((\sigma^\alpha(N_k, k), Y), N_k)$$

What we have defined just now as an assignment is different in various respects from assignments that are usual in the standard practice in logic.

First, an assignment in logic is global in the sense that it is defined for an entire language, while our assignment is local in the sense that it is defined for each particular indeterminate noun phrase. As a result, our assignment is for a finite number of different expressions, more precisely, different occurrences of expressions, and hence, it constitutes a finite list of pairs, while an assignment in logic is usually defined for an infinite number of variables and it constitutes an infinite list. This does not necessarily mean, however, that the total number of assignments for a given indeterminate noun phrase $\alpha$ is finite. For, if there are infinitely many objects in one of the domains of an indeterminate term that occurs in $\alpha$, then it gives rise to infinitely many different assignments for $\alpha$.

Yet this might not be an essential difference, because even in the case of standard logical practice it is possible to define an assignment for each sentence which we wish to give a semantic account, so long as we can ignore the relation between a number of different sentences that constitute a single discourse. As a sentence is finite in length, the number of variables that occur in it must be finite, and hence, it would be enough to consider only assignments for these variables.

A more essential and logically more interesting difference between them is that in the case of standard logic a relation between assignments which is called “variant relation” plays an important role to give an account of quantification. Two assignments are called variants on the $n$-th place of each other, when they are the same except what is assigned to the $n$-th variable, $v_n$. This relation is important because it makes possible to change a value for a specific variable without changing a value for any other variable, and this is essential to give an account of multiple quantification. Of course, we can also define a similar relation between our assignments to indeterminate noun phrases, but there is no point of doing this here\textsuperscript{5}.

The sort of multiple quantification which modern logic gave the first satisfactory account is possible only when different variables can be quantified successively by different quantifiers.

$$\forall x \exists y F(x, y)$$

\textsuperscript{5} We will see in Chapter 8, however, a variant relation is useful in giving an account of entailment between interrogative sentences.
is a typical example of such multiple quantification. We may regard it as a formula that has been constructed in stages.

1. $F_{xy}$
2. $\exists y F_{xy}$
3. $\forall x \exists y F_{xy}$

At stage 2, the variable “$y$” is bound by existential quantifier, but the other variable “$x$” is still free, and it is bound only at stage 3. Variant relation between assignments is needed to account for such successive quantifications.

The same process is also observed in a natural language, including Japanese. Consider a sentence

(83) Dono sensei mo kodomo o oshieta
which teacher ∀ child(ren) ACC taught
(Each teacher taught a child / children.)

“Kodomo” in (83) may be definite and (83) may mean that each teacher taught a certain particular child (or group of children), but here, I take up a reading which reads “kodomo” as indefinite. Then, the truth condition of (83) is derived through a process which is very similar to the stages 1–3.

1. $\text{Val}(⟨X, Y⟩, "oshieta")$
2. $\exists Y[\text{Val}(Y, "kodomo") \land \text{Val}(⟨X, Y⟩, "oshieta")])$
3. $\forall X[\text{Val}(X, "dono sensei") \rightarrow \exists Y[\text{Val}(Y, "kodomo") \land \text{Val}(⟨X, Y⟩, "oshieta")])$

You can see that these three stages correspond exactly to the stages above for a logical formula. If we wish to give a semantic account of our metalanguage, which is a language of plural logic with some vocabulary from natural language, we will have recourse to the assignments and variant relation among them as it is in the standard logic. It is because those cases of multiple quantification which are found both in logic and natural language are realized by successive applications of quantifiers.

In the case of quantification by “dono + N”, it never happens that one application of quantifier “mo” or “ka” leaves free some of indeterminate terms that occur in its restriction to be bound by succeeding applications of quantifiers. Take a sentence which is similar to (70) but importantly different in one small detail.

(84) Dono kodomo o oshieta dono sensei mo kita
which child ACC taught which teacher came
∀
There are two indeterminate terms “dono kodomo” and “dono sensei” in the restriction of a quantifier “mo”, and both of them are bound by the single occurrence of the quantifier “mo”. There is no way for one of them are left free. If there is a free occurrence of an indeterminate term within the restriction of “mo”, it is to be bound by “mo” no matter what it is. It is as if “mo” closes off any of indeterminate terms which occur free in its restriction from the rest of the sentence, and this is the reason why our assignment can be such a local affair as is limited to one particular indeterminate noun phrase.

We might say that a quantifier “mo” has two aspects, as it were, an inward one and an outward one. Its inward aspect consists in binding indeterminate terms that occur free in the restriction of “mo”. Its outward aspect consists in relating the semantic values of the restriction to those of a predicate that comes after “mo”. These two aspects coincide when the restriction of “mo” is an indeterminate term like “dono + N” and “dare”. Consider

(61) Dono kodomo mo waratta.
    which child ∀ laughed

(Every child laughed.)

On one hand “mo” binds “dono kodomo”, and on the other, it universally quantifies on individual children which are semantic values of the restriction. It is essential for an understanding of “mo” (and “ka”) to recognize that it has such a double function.

5.2 Presupposition of the non-emptiness of domain

There is one important point we need to be clear about the above definition of an assignment for an indeterminate noun phrase.

It concerns with a possibility that one of $N_k$’s, say $N_i$, might be empty, that is, might have an empty extension. $N_i$ is either a relational noun like “oya” (parent) or a non-relational noun like “kodomo” (child). Let us consider first the case where $N_i$ is a non-relational noun. If a non-relational noun $N_i$ is empty, then no individual will be among its semantic values. An assignment for $\alpha$ should pair with “(dono $N_i$)” an individual which is a semantic value of $N_i$, but there is no such individual in this situation. This means that there is no assignment for $\alpha$, either.

Turning to the case in which $N_i$ is a relational noun, we notice that the definition requires an assignment $\sigma$ for $\alpha$ should pair with “(dono $N_i$)” an

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6 Of course, “kodomo” (and “child”) has also a relational sense. But, I am concerned here only with its non-relational use. Cf. [Iida 2013].
individual which has the relation $R_i$ expressed by $N_i$ with some individuals. Hence, if there is no individual which has $R_i$ to any individuals, there is no assignment for $\alpha$.

I believe this is a necessary consequence of construing a phrase of the form “dono + N” as a sorted variable, unless we make our logic free and allow quantification over an empty domain. According to the usual or standard conception of quantification, a variable ranges over a domain that is not empty. The same thing should be true with a sorted variable, too. In both cases, if there is nothing in the domain, there is nothing that can be paired with a variable by an assignment. Thus, it is presupposed that the domain of quantification is not empty.

If we are working with a single domain and an unsorted variable, it is a very remote possibility that the domain turns out to be empty. But, if we are working with various sorted variables that correspond to small domains that may change depending on the context, it may easily happen that one of these small domains is empty. For example, consider the following sentence which is a variation of (65).

(85) Dono roku-sai-ji ga kaita tegami mo todoita.

six-year-child NOM wrote letter(s) ∀ arrived

(A letter / Letters that any six-year old wrote has arrived.)

We can easily imagine that (85) may be uttered by a person who is mistaken about the children she intends to talk about and there are no six-year olds in the relevant domain. If we know that the speaker is talking under such a misconception, how do we interpret her utterance of (85)? The likeliest way would be to interpret her to have uttered some other sentence which is close to (85) but does not reflect her mistake about the age of children, that is, some Japanese sentence to the effect that any letter that any child in such-a-such age group wrote has arrived. In other words, we do not try to determine whether (85) is true or false, but give it up as hopeless for a truth value evaluation and look for some related sentence that can be evaluated for its truth.

This suggests that we regard the sentence (85) uttered in such a circumstance as lacking a truth value. What causes this? It must be the fact that a sorted variable “dono roku-sai-ji” occurring in (85) lacks any semantical value. Conversely, when the utterance of (85) makes a true or false assertion, it is presupposed that there are some six-year olds in the domain.

In general, the use of an indeterminate noun phrase “dono + N” presupposes that there are some individuals in the domain which are among the semantic values of N, if N is a non-relational noun. I assume that if N is a relational noun like “oya” (parent) then the presupposition which is accompanied with the use of “dono + N” is that there is something which stand to some things in the relation expressed by N. This is the weakest condition for an assignment for a relational noun to be possible, and some might argue that a stronger condition
is required as the presupposition. In both cases, if the presupposition is not satisfied, then “dono + N” does not have any semantic value and any sentence which contains it lacks a truth value.

In this respect, the present account of “dono + N” is importantly different from the account we gave in the previous chapter, namely, one that construes “dono” as a singularizing predicate. According to that account, if there are no six-year olds, then that fact makes false the antecedent within a universal quantifier which is supplied by “mo” and that, in turn, makes the whole sentence trivially true.

(85) becomes also trivially true when there are six-year children and they wrote no letters. Some might think that this is a very unintuitive result, and for that reason, argue that in general it is a presupposition of a universal sentence “α mo φ” that its restriction α is not empty.

In fact, it is now a dominant opinion among linguists that many of the quantifiers in natural language are presuppositional in the sense that they have a presupposition whose unfulfillment leads to a truth value gap. The most well-known case is that of English “the”; starting from P.F.Strawson’s seminal paper “On Referring” (1950), many philosophers and linguists have argued that if a singular noun phrase beginning with “the” such as “the present King of France” does not have a semantic value if there is nothing corresponds to the description, and hence, that a sentence containing it is neither true nor false.

A similar claim about Japanese “mo” is that if its restriction is not satisfied by anything, then a universal statement formed by “mo” is neither true nor false.

Let us see whether this is true. We can test for presupposition by embedding a sentence which is claimed to have it in a variety of sentential contexts. Take (58) as an example.

(58) Dono kodomo ga dashita tegami mo todoita.
    which child NOM sent letter ∀ arrived
    (Letters sent by any child arrived.)

Now there are two candidates for presupposition this sentence might have. They are both existential presuppositions.

(86) Kodomo ga iru.
    child NOM exist
    (There are children.)

(87) Kodomo ga dashita tegami ga aru.
    child NOM sent letter NOM exist
(There are letters sent by children.)

Let us embed (58) into those sentential contexts which are thought to provide us tests for presupposition and see how (86) and (87) fare with the new environment.

(88) Dono kodomo ga dashita tegami mo todoita which child NOM sent letter ∀ arrived
to-iu-koto wa nai that TOP not
(86) It is not the case that letters sent by any child arrived.

(89) Dono kodomo ga dashita tegami mo todoita which child NOM sent letter ∀ arrived
to-iu no-naraba odoroki-da. that if surprising
(87) If letters sent by any child arrived, it is surprising.

(90) Tabun dono kodomo ga dashita tegami mo Probably which child NOM sent letter ∀ arrived
todoita.
(88) Probably letters sent by any child arrived.

(91) Dono kodomo ga dashita tegami mo todoita which child NOM sent letter ∀ arrived
to Taro wa omotte-iru. that TOP think
(90) Taro thinks that letters sent by any child arrived.

I think it is clear that we can infer (86) from each of (88)−(91). There is no question that the use of “dono + N” carries the presupposition that there is something that is N.

Is it the same with (87)? I might judge that (87) does not follow from (91). For, Taro may think that letters sent by any child arrived, even though there is no letter sent by any child in reality. But, I am not sure that (87) does not follow from (90), either. Is it possible to add that there might not be any letter sent by any child after uttering (90)? How about (88) and (89)? On one hand, it seems reasonable to infer (87) from either of them. But, on the other, it seems possible to continue either of them with the following statement.

(92) Datte dono kodomo mo tegami o dasa-nakka-ta. For which child ∀ letter ACC send-NOT-PAST
(For, no child had sent a letter/letters.)
This gives some support that (87) does not follow from (88) nor (89). But, this is far from decisive.

I suppose that we are facing a situation in which we had better explore a number of possible strategies to see what consequences each strategy involves. I consider here two strategies; one is logically conservative, and the other one is more on the radical side.

According to the first strategy, namely, a conservative one, (87) is not a presupposition of (58), but a consequence of some pragmatic inference. If you utter (58), and it turns out that no child wrote a letter, the statement itself will be true. But you will not be praised for making a true statement, but will be censured for making a misleading statement; you have made a statement which is pointless, because it is trivially true.

This strategy is logically conservative because it continues to maintain that even if the restriction of “mo” has nothing as its semantic value, the universal statement itself has a truth value, namely, truth. The other strategy denies this; if there is nothing which is a semantic value of the restriction of “mo”, then the whole statement is neither true nor false, namely, it lacks a truth value. The existence of some things which satisfy the restriction of “mo” is not a pragmatically derived consequence, but a presupposition that is built in the semantics of the universal quantifier “mo”. The latter strategy’s admission of truth value gap forces us to revise our logic, and therefore, it is not logically conservative.

It is important, however, to remember that this discussion about whether “mo” has presupposition or not is independent of our claim that a sentence containing a phrase “dono + N” has a presupposition that N is not empty. The reason why “dono + N” is thought to have such a presupposition is that it is construed as a sorted variable and that the domain of a variable should not be empty. If “mo” is to have a presupposition, then what would be its source? It could not be the same as that for the presupposition “dono + N” carries, because the restriction of “mo” is not necessarily a sorted variable\textsuperscript{10}.

Whether we think “mo” carries the existence presupposition or not, we suppose that the use of an indeterminate noun phrase “dono + N” gives rise to a presupposition and that the whole sentence would lack a truth value if this presupposition is not satisfied. Hence, to this extent, we have already departed from classical logic.

This means that a semantic account of Japanese sentences should be at least two-dimensional; it should assign to each Japanese sentence both its truth condition and its presuppositions. I leave, however, the problem of realizing such a multi-dimensional semantic account for a research in the future.

\textsuperscript{10} It can be so, as in a case where an indeterminate term “dono + N” constitutes the whole restriction of “mo”.
5.3 Semantics of “mo” in terms of assignments

In order to see how our assignments work, let us take up (82). In this sentence, “mo” appears only once, and its restriction is this indeterminate noun phrase

(l) dono kodomo ga dono kodomo ni
dashita tegami

which child NOM which child DAT
sent letter(s)

(Letters which child sent to which child)

In this, there are two occurrences of the same indeterminate term “dono kodomo” (which child), and both of them are free. Each assignment \( \sigma^{(l)} \), namely, assignments for (l) assigns individual children to these two occurrences of “dono kodomo”. If there are three children in all, say, Hanako, Taro, and Jiro, then there are \( 3 \times 3 \) different assignments for (l). Note that the use of “dono kodomo” generates the presupposition that there are some children in the domain.

Let us calculate the semantic values of (l), which are given relative to an assignment \( \sigma^{(l)} \).

1. \[
\text{Val}(X, "(((dono kodomo) ga) (((dono kodomo) ni) dashita)) tegami"), \sigma^{(l)}
\]

2. \[
\text{Val}(X, "(((dono kodomo) ga) (((dono kodomo) ni) dashita)", \sigma^{(l)})
\wedge \text{Val}(X, "tegami", \sigma^{(l)})
\]

2 is obtained from 1 by a relativised version of Axiom of noun modifying predicate we have met in the previous chapter. A general policy is to relativise the already introduced axioms to an assignment, if they should be applied to an expression which contains a free occurrence of an indeterminate noun phrase, and drop the reference to an assignment, if the expression to be given a semantic value no longer contains any free occurrence of an indeterminate noun phrase. Thus, we can change the second conjunct of 2 to “Val(X, “tegami”).”

The first conjunct of 2 is unpacked in this way.

3. \[
\exists Y [\text{Val}(Y, "((dono kodomo)", \sigma^{(l)}) \wedge \text{Val}((Y, X), "(((dono kodomo) ni) dashita)"), \sigma^{(l)}]
\]

Noting the order of the occurrences of the same indeterminate term “(dono kodomo)”, the first conjunct within “\( \exists Y \)” of 3 becomes this.

4. \( Y \equiv \sigma^{(l)}("kodomo", 1) \)

\(^{11}\) If we adopt a more natural reading of (l) and no child wrote a letter to itself, then we need to consider only \( 3 \times 2 \) assignments.
On the other hand, the second conjunct becomes

5. $\exists Z [\text{Val}(Z, \textbf{“(dono kodomo)”}, \sigma^{(l)}) \land \text{Val}(\langle Y, Z, X \rangle, \textbf{“dashita”})]$, 

and then,

6. $\exists Z [Z \equiv \sigma^{(l)}(\textbf{“kodomo”}, 2) \land \text{Val}(\langle Y, Z, X \rangle, \textbf{“dashita”})]$, 

Thus, combining 4 with 6 and changing plural variables into singular ones wherever possible, 3 as a whole becomes

7. $\exists y [y = \sigma^{(l)}(\textbf{“kodomo”}, 1) \land \exists z [z = \sigma^{(l)}(\textbf{“kodomo”}, 2) \land \text{Val}(\langle y, z, X \rangle, \textbf{“dashita”})]]$. 

This can be simplified to

8. $\text{Val}(\langle \sigma^{(l)}(\textbf{“kodomo”}, 1), \sigma^{(l)}(\textbf{“kodomo”}, 2), X \rangle, \textbf{“dashita”})$. 

Lastly, we get the following specification of the semantic values of (I).

9. $\text{Val}(\langle \sigma^{(l)}(\textbf{“kodomo”}, 1), \sigma^{(l)}(\textbf{“kodomo”}, 2), X \rangle, \textbf{“dashita”}) \land \text{Val}(X, \textbf{“tegami”})$ 

This means literally that

$X$ are letters which an object assigned by $\sigma^{(l)}$ to the first occurrence of “dono kodomo” in (I) sent to an object assigned by $\sigma^{(l)}$ to the second occurrence of “dono kodomo” in (I).

It is much better, however, to adopt Japanese as our metalanguage and read 9 in this way.

$kono_1$ kodomo ga $kono_2$ kodomo ni dashita tegami

(this$_1$ child NOM this$_2$ child DAT sent letter(s))

(letter(s) this$_1$ child sent to this$_2$ child)

Assignments are sometimes compared to successive pointings. You point to one child in the domain saying “kono kodomo ga” (this child NOM), and then point to another child (or, the same child) saying “kono kodomo ni” (this child DAT). With the rest of the phrase, you have characterized a certain letters which were supposed to be sent from the first child to the second child. There might be such letters, but it is equally possible that there are none.

Incidentally, the fact that we may substitute “kono kodomo” for “dono kodomo” in this way suggests that the concept of assignment may turn out to be a key to locate “dono + N” within the series of the so-called “ko-so-a-do” phrases, namely, “kono + N”, “sono + N”, “ano + N” and “dono + N”. We

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will discuss how our semantics of “dono + N” can be fitted into the various phenomena including demonstratives in chapter 9.

Returning to our present example, let us consider what happens if we form a universally quantified sentence with the indeterminate noun phrase (l) as (82), which has the form

\[(82) \quad (l) \quad \text{mo todoita.} \]

\[\forall \quad \text{arrived}\]

What it says is roughly\(^{12}\) that if there are letters which some child wrote to another (possibly the same) child then they have arrived.

In light of our new account of the semantics of an indeterminate noun phrase, the semantics of a universal quantifier particle “mo” should be revised\(^{13}\).

**Axiom of a universal quantifier “mo” (second version)**

Let \(\alpha\) be an indeterminate noun phrase and suppose that there are assignments for \(\alpha\).

(i) If \(\phi\) is a one-place predicate, then “(\(\alpha\ \text{mo}\) \(\phi\))” is true if and only if

\[\forall X[\exists \sigma^\alpha \text{Val}(X, \alpha, \sigma^\alpha) \Rightarrow \text{Val}(X, \phi)].\]

(ii) If \(\phi\) is a two-place predicate, then \(\text{Val}(X, “(\alpha \ [\text{ga}] \ \phi)”\)) if and only if

\[\forall Y[\exists \sigma^\alpha \text{Val}(Y, \alpha, \sigma^\alpha) \Rightarrow \text{Val}((Y, X), \phi)],\]

and \(\text{Val}(X, “(\alpha \ [\text{o}] \ \text{mo}) \ \phi)”\) if and only if

\(^{12}\) Why “roughly”? It is because (82) does not imply that all letters some child wrote to some child have arrived. It is enough that for each pair of children some of the letters the one wrote to the other have arrived. This is one of the main topics of the next chapter.

\(^{13}\) The semantics of an existential quantifier particle “ka” should also be revised, so that it is given in terms of assignments. Let us consider a case with \(N\) a common noun and \(\phi\) a one-place predicate. This can be easily adapted for other cases. Let \(D\) be an indeterminate term of the form “(dono \(N))”. If we reformulate the axiom in the previous chapter in terms of assignments, we have

(i) “((D \ ka) \ \text{ga}) \ \phi)” is true if and only if

\[\exists X[\exists \sigma^D \text{Val}(X, D, \sigma^D) \wedge \text{Val}(X, \phi)].\]

But, as

\[\text{Val}(X, D, \sigma^D) \Leftrightarrow X \equiv \sigma^D(N, 1)\]

The right hand side of (i) can be simplified to

(ii) \(\exists \sigma^D \text{Val}(\sigma^D(N, 1), \phi)\)

We should suppose that \(N\) is non-empty. Otherwise, the presupposition that there are assignments for “(dono \(N))” would be false. Under this supposition, (ii) is equivalent to

(iii) \(\exists x[\text{Val}(x, N) \wedge \text{Val}(x, \phi)]\)

This, however, should not be part of the axiom itself, but its consequence.
∀Y[∃σ^αVal(Y, α, σ^α) → Val((X, Y), ϕ)].

In case (i), "(α mo) ϕ" might be "(α mo [gal]) ϕ" as well as "(α [o] mo) ϕ".

Please note that in the above "σ^α" is a sorted mata-variable ranging over assignments for an indeterminate noun phrase α. A sorted meta-variable "σ^α" represents a number of indeterminate noun phrases of the form "dono N_k", which may be regarded as object language sorted variables. Object language presuppositions that any of the domain of "dono N_k" is not empty becomes a claim that the domain of meta-variable "σ^α" is not empty, namely, there exist assignments for α.

Thus, the axiom above which is formulated in terms of the quantification over assignments for α requires the presupposition that there exist assignments for α. If we know what α is, we will know what this presupposition amounts to.

Suppose that, as in the definition of an assignment, D_1, D_2, . . . , D_n are all the occurrences of indeterminate noun phrases of the form "dono N_k" (1 ≤ k ≤ n) which are free in α. Then, in order for α to have semantic values, it is necessary that, for each k (1 ≤ k ≤ n),

if N_k is a non-relational noun, then ∃xVal(x, N_k).

if N_k is a relational noun, then ∃x∃YVal((x, Y), N_k).

These should be among the presuppositions of a universally quantified sentence with "mo" which has α as its restriction.

As "σ^α" is a variable for assignments for α, "∃σ^αVal(X, α, σ^α)" means that there is an assignment for α relative to which X are semantic values of α. Thus, the above axiom says that, any things that are α relative to a suitable assignment have the property ϕ, or the property of standing in the relation ϕ.

You can also see the dual aspect of "mo" from this axiom. It shows that two sorts of quantification are involved in the semantics of "mo". They are quantification over assignments and that over semantic values common to α and ϕ. The former represents the inward aspect of "mo" and the latter its outward aspect. The former’s scope is limited to the antecedent, while the latter’s scope extends to the entire sentence or open sentence.

The best way to understand the axiom, however, is to see how it works with some examples. Let us start with a sentence simpler than (82). Take the sentence (64).

(64) Dono kodomo no oya mo kita.

(child(ren) GEN parent(s) ∀ came)

(Any child’s parent(s) came.)

A formal representation of this sentence is
Let \((m)\) be an abbreviation for the indeterminate noun phrase “\(((\text{dono kodomo})\ \text{no})\ \text{oya})\). By the above axiom of “mo”, the truth condition of this sentence is given by the following, provided that assignments for \((m)\) exist, or equivalently,

\[\exists x \text{Val}(x, \text{"kodomo").}\]

1. \(\forall X[\exists \sigma(m)\text{Val}(X, (m), \sigma^{(m)}) \rightarrow \text{Val}(X, \text{"kita")}]\)

Let us unpack the part “\(\text{Val}(X, (m), \sigma^{(m)})\)”.

2. \(\text{Val}(X, (((\text{dono kodomo})\ \text{no})\ \text{oya}), \sigma^{(m)})\)

3. \(\exists Y[\text{Val}(Y, (((\text{dono kodomo})\ \text{no})\ \text{oya}), \sigma^{(m)}) \land \text{Val}((X, Y), \text{"oya")}]\)

4. \(\exists Y[Y = \sigma^{(m)}(\text{"kodomo"}, 1) \land \text{Val}((X, Y), \text{"oya")}]\)

5. \(\text{Val}((X, \sigma^{(m)}(\text{"kodomo"}, 1)), \text{"oya")}\)

Putting this back to 1, we get

6. \(\forall X[\exists \sigma^{(m)}\text{Val}((X, \sigma^{(m)}(\text{"kodomo"}, 1)), \text{"oya")} \rightarrow \text{Val}(X, \text{"kita")}]\)

I will show that 6 is equivalent to 7, provided that assignments for \((m)\) exist.

7. \(\forall X[\exists y[\text{Val}(y, \text{"kodomo")} \land \text{Val}((X, y), \text{"oya")}] \rightarrow \text{Val}(X, \text{"kita")}]\)

In order to show that 6 and 7 are equivalent under the above presupposition, it is sufficient to show the following 8 and 9 are equivalent.

8. \(\exists \sigma^{(m)}\text{Val}((X, \sigma^{(m)}(\text{"kodomo"}, 1)), \text{"oya")}\)

9. \(\exists y[\text{Val}(y, \text{"kodomo")} \land \text{Val}((X, y), \text{"oya")}]\]

[8 \rightarrow 9] Suppose 8 is true for an arbitrary \(X\). Then, there is an assignment \(\sigma_0^{(m)}\) such that

10. \(\text{Val}((X, \sigma_0^{(m)}(\text{"kodomo"}, 1)), \text{"oya")}\).

As \(\sigma_0^{(m)}\) is an assignment for \((m)\), by the definition of an assignment for an indeterminate noun phrase, in particular, by (C1),

11. \(\text{Val}(\sigma_0^{(m)}(\text{"kodomo"}, 1), \text{"kodomo")}\)

From 10 and 11, we get 9 by existential generalization.

[9 \rightarrow 8] Suppose 9 is true for an arbitrary \(X\). Then, there is a \(y_0\) such that
12. Val\( (y_0, \text{“kodomo”}) \) and
13. Val\( ((X, y_0), \text{“oya”}) \).

From 12, there is an assignment \( \sigma_0^{(m)} \) for \text{“dono kodomo no oya”} such that
14. \( y_0 = \sigma_0^{(m)}(\text{“kodomo”}, 1) \).

Moreover, as Val\( ((X, y_0), \text{“oya”}) \), we have
15. Val\( ((X, \sigma_0^{(m)}(\text{“kodomo”}, 1)), \text{“oya”}) \)

Remembering that \( \sigma_0^{(m)} \) is one of \( \sigma^{(m)} \), we have 8.

The fact that 8 and 9 are equivalent under the assumption that there are some things which are semantic values of \text{“kodomo”} suggests that we can freely go from an assignment for an indeterminate phrase \( \alpha \) to certain individuals which function as a sort of parameters for the semantic values of \( \alpha \) and vice versa, provided that any indeterminate term that occurs freely in \( \alpha \) has a non-empty extension.

The last provision is essential; without it, 8 is not equivalent to 9, and hence, 6 is not equivalent to 7, either. If there is no child in the domain, 8 is undefined because there is nothing \( \sigma^{(m)} \) to range over, and hence, 6 will lack a truth value, while 7 will be true because of the falsity of the antecedent.

When the existence presupposition of the sorted variable \text{“dono kodomo”}, namely, the non-emptiness of the sorted domain, is satisfied, 6 is equivalent to 7. But, as we have seen in our previous account which construed \text{“dono”} as a singularizing predicate, the existential quantification whose scope is the antecedent of 7 can be turned equivalently into universal quantification whose scope is the whole sentence like the following.

16. \( \forall X \forall y [(\text{Val}(y, \text{“kodomo”}) \land \text{Val}((X, y), \text{“oya”})) \rightarrow \text{Val}(X, \text{“kita”})] \)

It is just the same with 6, which can be reformulated thus.

17. \( \forall X \forall \sigma^{(m)} [(\text{Val}((X, \sigma^{(m)}(\text{“kodomo”}, 1)), \text{“oya”})) \rightarrow \text{Val}(X, \text{“kita”})] \)

Thus, the present account gives (64) the same truth condition as the previous one so far as the existential presupposition relating to \text{“dono kodomo”} is satisfied. But, when the presupposition fails, the difference between two accounts becomes clear; for, then (64) will be neither true nor false according to the present account and hence, its truth condition is no longer expressed by 16.

Next, let us see how the present account handles the simplest cases like (61), namely a “mo” sentence the restriction of which is just a simple indeterminate noun phrase.
(61) Dono kodomo mo waratta.
   child(ren) ∀ laughed

(Every child laughed.)

Suppose that \( \sigma \) is a meta-variable ranging over assignments for “(dono kodomo)”. By the axiom of “mo”,

1. “((dono kodomo) mo) waratta” is true

is equivalent to

2. \( \forall X [\exists \sigma \text{Val}(X, “(dono kodomo)”, \sigma) \rightarrow \text{Val}(X, “waratta”)] \).

2 becomes

3. \( \forall X [\exists \sigma X = \sigma(“kodomo”, 1) \rightarrow \text{Val}(X, “waratta”)] \).

Changing a plural variable \( X \) to a singular variable \( x \), we have

4. \( \forall x [\exists x = \sigma(“kodomo”, 1) \rightarrow \text{Val}(x, “waratta”)] \).

Although it might not be obvious, the antecedent of the conditional within the universal quantifier, namely,

5. \( \exists x = \sigma(“kodomo”, 1) \)

is equivalent to

6. \( \text{Val}(x, “kodomo”) \).

Let us see how this will be shown.

[5 \rightarrow 6] Suppose 5 for an arbitrary \( x \). Then, there is an assignment \( \sigma_0 \) for “(dono kodomo)” such that

7. \( x = \sigma_0(“kodomo”, 1) \)

By the definition of an assignment for “(dono kodomo)”, in particular, by (C1),

8. \( \text{Val}(\sigma_0(“kodomo”, 1), “kodomo”) \)

But then, 6 is true. ■

[6 \rightarrow 5] Conversely, suppose 6 is true for an arbitrary \( x \). Then, by the definition of an assignment for “(dono kodomo)”, there is some assignment \( \sigma_0 \) for “(dono kodomo)” which assigns \( x \) to the first occurrence of “(dono kodomo)” in “(dono kodomo)”, that is,
\[ x = \sigma_0(\text{"kodomo"}, 1) \]

Hence, by existential generalization, we get 5.

Thus, it turns out that 4 is equivalent to

9. \( \forall x[\text{Val}(x, \text{"kodomo")} \rightarrow \text{Val}(x, \text{"waratta")}] \),

provided that there are some children in the domain. If it happens that there are no children in the domain, 4 and 9 would not be equivalent, because in that case 5 would be undefined, and hence, 4 would be neither true nor false, but 9 would be true because its antecedent would be false for any \( x \).

So far, we have worked under a version of an axiom of “mo” which recognizes only presuppositions carried by indeterminate terms of the form “dono + N”. We should also consider a version of an axiom of “mo” which incorporates the idea that “mo” carries the presupposition that its restriction is not empty. It is not difficult to state an alternative axiom itself. The difficulty is how to revise our logic so that it allows a truth value gap in a coherent way. As I have no new idea about this problem, I am content with simply stating the axiom.

**Axiom of a universal quantifier “mo”** (alternative version)

Let \( \alpha \) be an indeterminate noun phrase and suppose that there are assignments for \( \alpha \).

(i) Let \( \phi \) be a one-place predicate.
   
   If \( \exists X \exists \sigma \exists \alpha \exists X, \alpha, \sigma \rightarrow \text{Val}(X, \phi) \),
   
   then “(\( \alpha \text{ mo} \) \( \phi \))” is true if and only if
   
   \( \forall X[\exists \sigma \exists \alpha \exists \alpha \rightarrow \text{Val}(X, \phi)] \),

   otherwise, “(\( \alpha \text{ mo} \) \( \phi \))” is neither true nor false.

(ii) Let \( \phi \) be a two-place predicate.
   
   If \( \exists Y \exists \sigma \exists \alpha \exists Y, \alpha, \sigma \rightarrow \text{Val}(Y, \langle X, Y \rangle, \phi) \),
   
   then Val(X, “(\( \alpha \text{ mo [ga]} \) \( \phi \))”) if and only if
   
   \( \forall Y[\exists \sigma \exists \alpha \exists \alpha \rightarrow \text{Val}(\langle Y, X \rangle, \phi)] \),

   and Val(X, “(\( \alpha \text{ [o] mo} \) \( \phi \))”) if and only if
   
   \( \forall Y[\exists \sigma \exists \alpha \exists \alpha \rightarrow \text{Val}(\langle X, Y \rangle, \phi)] \),

   otherwise, Val(X, “(\( \alpha \text{ mo [ga]} \) \( \phi \))” and Val(X, “(\( \alpha \text{ [o] mo} \) \( \phi \))”) are undefined.

In case (i), “(\( \alpha \text{ mo} \) \( \phi \))” might be “(\( \alpha \text{ mo [ga]} \) \( \phi \))” as well as “(\( \alpha \text{ [o] mo} \) \( \phi \))”.
Let us note the two versions of an axiom of “mo” coincide when the restriction of “mo” is just a simple indeterminate phrase as is the case with (61). It is because in this case semantic values of the restriction are just those objects which are assigned by assignments for the simple indeterminate phrase and their existence is guaranteed by the existence of assignments\textsuperscript{14}.

5.4 Presupposition again. Appropriateness of assignments

As the last example, let us consider a sentence which is similar to (64) but contains two “dono + N” phrases. This example is going to raise some interesting questions.

(93) Dono kodomo no dono oya mo kita.

child(ren) GEN parent(s) ∀ came

(Any parent of any child came.)

This sentence is formally represented as

\((((((dono kodomo) no) (dono oya)) mo) kita\)

The only difference from (64) is that “oya” is now part of an indeterminate term “dono oya”, and hence, we can proceed just as we did with (64) until a certain point.

Let “(n)” be an abbreviation for the indeterminate noun phrase “(((dono kodomo) no) (dono oya))”, and “σ” be a meta-variable ranging over assignments for (n). By the axiom of “mo”, the truth condition of this sentence is the following.

1. \(\forall X[\exists \sigma \text{Val}(X, (n), \sigma) \rightarrow \text{Val}(X, \text{“kita”})]\)

Let us unpack the part “\text{Val}(X, (n), \sigma)”.

2. \(\text{Val}(X, \text{“(((dono kodomo) no) (dono oya))”, \sigma})\)

3. \(\exists Y[\text{Val}(Y, \text{“(dono kodomo)”, \sigma}) \land \text{Val}((X, Y), \text{“(dono oya)”, \sigma})]\)

4. \(\exists Y[Y = \sigma(\text{“kodomo”, 1}) \land \text{Val}((X, Y), \text{“(dono oya)”, \sigma})]\)

Until this point, we have proceeded just as we did with (64) except the relativization to an assignment \(\sigma\) for “dono oya”. From now on, the difference owing to the presence of the second indeterminate term “dono oya” becomes important. We should unpack the second conjunct inside “\(\exists Y\)”, namely,

\textsuperscript{14} For the same reason, there is no need to consider an alternative version of an axiom of “ka”. It is because “ka” only attaches to an indeterminate term.
5. Val(⟨X, Y⟩, “(dono oya)”, σ)

6. X = σ(“oya”, 2) ∧ Val(⟨X, Y⟩, “oya”).

Putting 6 back into 4, and 4 into 1 in turn, while changing a plural variable to a singular one, we have

7. ∀x[∃σ∃y[y = σ(“kodomo”, 1) ∧ x = σ(“oya”, 2) ∧ Val(⟨x, y⟩, “oya”)] → Val(x, “kita”)]

As before, we can claim that 7 is equivalent to the following 8 under the supposition that two sorted variables “dono kodomo” and “dono oya” have non-empty domains.

8. ∀x[∃y[Val(y, “kodomo”) ∧ Val(⟨x, y⟩, “oya”) → Val(x, “kita”)]]

The equivalence between 7 and 8 under the supposition is again shown by the equivalence between their antecedents for an arbitrary x.

[7 → 8] Suppose that

9. ∃σ∃y[y = σ(“kodomo”, 1) ∧ x = σ(“oya”, 2) ∧ Val(⟨x, y⟩, “oya”)]

Let σ₀ and y₀ be such σ and y which are said to exist by 9. Then, for an arbitrary x,

10. σ₀ is an assignment for (n),

11. y₀ = σ₀(“kodomo”, 1),

12. x = σ₀(“oya”, 2), and

13. Val(⟨x, y₀⟩, “oya”).

By the definition of an assignment for (n), in particular, by (C1), we get

14. Val(y₀, “kodomo”)

from 11. From this and 13, we have

15. ∃y[Val(y, “kodomo”) ∧ Val(⟨x, y⟩, “oya”)]

[8 → 7] Conversely, suppose 15 is true for an arbitrary x, and let y₀ be such y which is said to exist.

As Val(⟨x, y₀⟩, “oya”), it follows that

16. ∃YVal(⟨x, Y⟩, “oya”)
Moreover, Val\((y_0, \text{“kodomo”})\). Hence, by the definition of an assignment for \((n)\), there is an assignment \(\sigma_0\) for \((n)\) such that

16. \(y_0 = \sigma_0(\text{“kodomo”}, 1)\).

17. \(x = \sigma_0(\text{“oya”}, 2)\).

From 16 and 17, by existential generalization we have

18. \(\exists y[y = \sigma_0(\text{“kodomo”}, 1) \land x = \sigma_0(\text{“oya”}, 2)]\)

As \(\sigma_0\) is an assignment for \((n)\), we have 9.

Remember that (93) is the same as (64) except that (93) has as the restriction of “mo” an indeterminate noun phrase ending with an indeterminate term “dono oya” instead only “oya”. Let us see what truth conditions our present account gives for them. The truth condition of (64) that was derived in the previous section is this.

\[(64^\prime) \forall X[\exists y[\text{Val}(y, \text{“kodomo”}) \land \text{Val}(\langle X, y \rangle, \text{“oya”})] \rightarrow \text{Val}(X, \text{“kita”})]\]

On the other hand, the truth condition of (93) was given above as

\[(93^\prime) \forall x[\exists y[\text{Val}(y, \text{“kodomo”}) \land \text{Val}(\langle x, y \rangle, \text{“oya”})] \rightarrow \text{Val}(x, \text{“kita”})]\]

We notice that the only difference between (64’) and (93’) is that a plural variable “\(X\)” in the former is a singular “\(x\)” in the latter. This is expected, because the presence of “dono” gives rise to singular quantification. There is, however, a suspicion that this difference is only a superficial one. For, it may be argued that (64’) and (93’) are equivalent; if any parents of each child came, then each parent of each child came, and conversely, if each parent of each child came, then any parents of each child came. This is the problem we mentioned at the beginning of this chapter, and as I said there, it is the problem we are going to be concerned with in the next chapter.

Here, I would like to consider another problem. It is what presuppositions each of (64) and (93) has. If we stick to the “conservative” version of axiom of “mo”, it follows that (64’) and (93’) have different presuppositions. (93) has two presupposition, namely

\[(94) \exists x \text{Val}(x, \text{“kodomo”}), \text{ and}\]

\[(95) \exists x \exists Y \text{Val}(\langle x, Y \rangle, \text{“oya”}),\]

while (64) has only (94) as its presupposition.

The question whether (94) is the only presupposition of (64) is closely related to another question, namely, which axiom of “mo” is the right one, the “conservative” one or alternative one. Before discussing it, I would like to consider yet
another question, namely, whether (94) and (95) are all of the existence presuppositions of (93). This question is independent of the choice between different axioms of “mo”.

According to our definition of assignments for an indeterminate noun phrase, if one of the indeterminate term of the form “dono + N” has a relational noun as N, then what is required of an assignment σ is only that an object which is assigned by σ to “dono + N” stands in the relation denoted by N to some things. This is the reason why (95) is a presupposition of (93). “Dono oya” in (93), however, does not seem to denote just a parent of somebody; what it denotes is not anybody’s parent, but a parent of a child which is denoted by “dono kodomo”.

To see that (94) and (95) may not be all the presuppositions of (93), imagine a context in which there are some children and some adults. Further suppose that some of the adults are parents of somebody, but among them there are no parents of those children who are assumed to exist in the context. If (93) presupposes only (94) and (95), then (93) uttered in such a context would not lack a truth value, because (94) and (95) are both true in it; (93) would be trivially true, because there is no assignment which makes the antecedent true. A natural reaction in such a context, however, seems to say that if none of the children in question do not have parents, then the question of the truth of (93) does not arise at all, that is, (93) is neither true nor false.

This suggests that the trouble with our definition of assignment for α is that its assignments to each occurrence of an indeterminate term are independent from each other. Different occurrences of indeterminate terms in α may semantically depend on each other, and any assignment for α should reflect it. In the case of (93), the occurrence of “dono oya” (which parent) is dependent on the preceding occurrence of “dono kodomo” (which child), namely the reference of “dono oya” should be to a parent of the referent of “dono kodomo”; and hence, an assignment for “dono kodomo no dono oya” should capture this dependence.

How can we do this? I suppose the simplest way is to leave our definition of assignments as it is, and select from them some that satisfy the additional conditions which reflect the semantic dependence between indeterminate terms. In the present example, any appropriate assignments σ for “dono kodomo no dono oya” should satisfy the following condition as well.

\[ \sigma(\text{"oya"}, 2) \text{ is a parent of } \sigma(\text{"kodomo"}, 1), \text{ namely,} \]

(96) \text{Val}(\langle \sigma(\text{"oya"}, 2), \sigma(\text{"kodomo"}, 1) \rangle, \text{"oya"}).

It is necessary for such assignments to exist that there are two individuals in the relevant domains which stand in the parent relation. This is the presupposition for “dono kodomo no dono oya” to have a semantical value. Thus, the following should be counted as a presupposition of (93).
(97) $\exists x \exists y [\text{Val}(y, \text{"kodomo"}) \land \text{Val}((x, y), \text{"oya"})]$

You will notice that (94) and (95) both follow from (97), but (97) does not follow even from their conjunction.

Once we are aware that there may be additional conditions for assignments for an indeterminate noun phrase, we notice that such conditions are not necessarily concerned with the dependence of one indeterminate term on another. Consider this sentence, which is a variant of (58).

(98) Kodomo ga dashita dono tegami mo todoita.

(child(ren) NOM sent which letter ∀ arrived)

(Any letter sent by a/the child/children has arrived.)

As its English translation indicates it has at least two readings depending on whether “kodomo” is interpreted as definite or indefinite. Let us take up an indefinite reading.

Any assignment for “kodomo ga dashita dono tegami” (letter sent by a child/children) should assign a letter to an indeterminate term “dono tegami”, that is, if $\sigma$ is such an assignment, then it should be the case that

(99) $\text{Val}(\sigma(\text{"tegami"}, 1), \text{"tegami"})$.

This reflects the fact that “dono tegami” is a sorted variable that ranges over letters. But, is it true that the domain of “dono tegami” in (98) consists of all the letters in the context? Doesn’t it a more restricted one which consists of the letters sent by a child or children? If we answer yes to this, then we are to hold in general that an indeterminate term with a qualification is a sorted variable ranging over a domain further restricted by the qualification. The semantic dependence between indeterminate terms is just its special case.

In order to have a general characterization of semantic dependence that is found among the constituents of a complex indeterminate noun phrase, we should start with some syntactic matters.

Let us go back to the definition of an indeterminate noun phrase in §4.2. As it is concerned with a small fragment of Japanese, it covers only a very small part of indeterminate noun phrases in general. But, working with something like it helps us to see what should be done for a more general treatment.

If we look over that definition of an indeterminate noun phrase, we will notice that there are two ways of qualifying an indeterminate term. They are putting in front of “dono + N” either a noun phrase with a case particle “no”, or a one place predicate. So, we have this.

**Definition: a qualified indeterminate term**

An indeterminate noun phrase $\alpha$ contains an occurrence of a qualified indeterminate term “dono N” if and only if
(i) there is an indeterminate term “dono N” and a noun phrase $\beta$ such that

$$((\beta \text{ no}) \text{ (dono N)})$$

is a constituent of $\alpha$, or

(ii) there is an indeterminate term “dono N” and a one place predicate $\phi$ such that

$$((\phi \text{ (dono N)})$$

is a constituent of $\alpha$.

I should add that several things are lacking for this to be a real definition. For one thing, we have not yet defined what a constituent of a phrase is, and for another, a general characterization of a one place predicate is missing. Yet, this “definition” gives us a general idea about what is involved.

If we were given a definition of a qualified indeterminate term, we could define when an assignment for an indeterminate noun phrase $\alpha$ is called appropriate.

**Definition: Appropriate assignment**

An assignment $\sigma$ for an indeterminate noun phrase $\alpha$ is an appropriate assignment for $\alpha$ when it satisfies the following condition.

(i) If $\alpha$ contains an occurrence of a qualified indeterminate term of the form

$$((\beta \text{ no}) \text{ (dono } N_k)),$$

where $\beta$ is a noun phrase and “(dono $N_k$)” is an n-th free occurrence of an indeterminate term in $\alpha$, then either of the following holds:

(a) if $N_k$ is a relational noun, then

$$\exists X[\text{Val}(X, \beta, \sigma) \land \text{Val}(\sigma(N_k, k), X), N_k]],$$

or

(b) if $N_k$ is a non-relational noun, then

$$\exists X[\text{Val}(X, \beta, \sigma) \land [\text{Val}(\sigma(N_k, k), N_k) \land RC(\sigma(N_k, k), X)],$$

where $RC$ is a relation determined by a given context $C^{15}$.  

(ii) If $\alpha$ contains an occurrence of a qualified indeterminate term of the form

$$((\phi \text{ (dono } N_k)),$$

15 See a footnote appended on an axiom of relational noun in §4.3.
where \( \phi \) is a one place predicate and \( \text{“(dono } N_k \text{)”} \) is an \( n \)-th free occurrence of an indeterminate term in \( \alpha \), then

\[
\text{Val}(\sigma(N_k, k), \phi, \sigma)
\]

Let us apply this first to “dono kodomo no dono oya” of (93) and then, to “kodomo ga dashita dono tegami” of (98).

Let \( \sigma \) be an assignment for “dono kodomo no dono oya”. As this phrase contains, or rather, is a qualified indeterminate term, and “oya” is a relational noun, the case (i–a) of the above definition applies. Hence, for \( \sigma \) to be an appropriate assignment for it, it must satisfy this.

\[
\exists X [\text{Val}(X, \text{“dono kodomo”}, \sigma) \land \text{Val}(\langle \sigma(\text{“oya”}, 2), X \rangle, \text{“oya”})]
\]

As “\( \text{Val}(X, \text{“dono kodomo”}, \sigma) \)” is equivalent to

\[
\exists X \text{Val}(X, \text{“dono kodomo”}, \sigma)
\]

(100) comes down to

\[
\text{Val}(\langle \sigma(\text{“oya”}, 2), \sigma(\text{“kodomo”}, 1) \rangle, \text{“oya”}).
\]

This is just the same as (96), which we claimed to be a presupposition of (93). As it was remarked there, (97) is a necessary and sufficient condition that an assignment that satisfies (96) to exist, and hence, (97) is a presupposition for “dono kodomo no dono oya”’s having semantic values.

Let us move to the next example. Suppose \( \sigma \) is an assignment for “kodomo ga dashita dono tegami”. This is again a qualified indeterminate term, and this time the case (ii) applies. For \( \sigma \) to be an appropriate assignment, it must satisfy the following condition.

\[
\text{Val}(\sigma(\text{“tegami”}, 1), \text{“(kodomo ga dashita)”}, \sigma)
\]

This is equivalent to

\[
\exists X [\text{Val}(X, \text{“kodomo”}) \land \text{Val}(\langle X, \sigma(\text{“tegami”}, 1) \rangle, \text{“dashita”})]
\]

This says that the object which is assigned by \( \sigma \) to “dono tegami” should be not only a letter but also a letter sent by some child or children, that is, that \( \sigma(\text{“tegami”}) \) should satisfy (103) as well as (99). And, for the existence of an assignment which satisfies (103), it is necessary and sufficient that the following is true.
Thus, (104) is a presupposition for an indeterminate noun phrase “dono kodomo ga dashita tegami” to have semantic values.

It is almost certain, however, that a supporter of an alternative axiom of “mo” would argue that there is no need to any of the above, because just the same results can be had by adopting the alternative axiom. According to it, (93) has a presupposition that the antecedent can be true, that is just what (97) says. Again, we can infer that (98) has (104) as its presupposition from this version of axiom of “mo”. No appeal to an appropriate assignment is necessary.

We can even form a conjecture like the following. This must be a conjecture because I have not checked whether it is true for any possible indeterminate noun phrase. It is very likely to be true, however, for a fragment like $J$ defined in §4.2.

**Conjecture**

For a sentence of the form “$(\alpha \text{ mo } \phi)$”, the following holds.

If $\alpha$ is not empty, that is, $\exists X \exists \sigma \text{ Val}(X,\alpha,\sigma)$, then there exist appropriate assignments for $\alpha$.

Of course, even though the conclusions are the same, the theoretical motivations behind them are different. The appeal to appropriate assignments is based on the idea that “dono” makes a sorted variable and that the domain of a variable should not be empty. In contrast, the adoption of an alternative axiom of “mo” is motivated by a claim that “mo” is one of the presuppositional quantifiers that are often found in a natural language. This difference of the motivations reflects again the dual aspect of the quantifier “mo”; some may think that quantification over assignments by means of a sorted variable “dono + N” generates presupposition, and some may think that a presuppositional quantifier “mo” generates presupposition.

It is not true that these two approaches reach always the same conclusion. It is clear that the converse of the above conjecture is not true. A case in point is (64), which I repeat here again.

(64) Dono kodomo no oya mo kita.
child(ren) GEN parent(s) ∀ came
(Any child’s parent(s) came.)

If we do not adopt the alternative axiom for “mo”, then (64) has only the existence of some children as its presupposition. In contrast, if we subscribe to
the view that “mo” is a presuppositional quantifier and adopt the alternative axiom, then (64) has a stronger presupposition that there are some children and their parents.

Even when one does not adopt the alternative axiom and thinks that the stronger presupposition does not result from the semantics of (64), one can still admit that (64) comes to have it through some pragmatic inference. It is very difficult, however, to judge whether the presupposition that seems to accompany (64) is generated by a pragmatic inference or a semantic machinery.

Although we have been wondering which of the opposing views is the correct one, this may not be the right perspective to see the situation. We should realize that these two views are not necessarily incompatible. As a matter of fact, if the above conjecture is true, then the alleged presuppositional character of “mo” implies the non-emptiness of the domains of sorted variables that appear in the restriction of “mo”. Thus, our question is not to choose between two exclusive options, but to choose between a stronger one and a weaker one.

In next chapter, I am going to argue that there is some ground for counting “mo” into presuppositional quantifiers. It does not mean that I will no longer hold that the non-emptiness of the domain of a sorted variable “dono + N” is a necessary condition for its having a semantic value. The above conjecture is very important, because if it is true it gives us a guarantee that all of the sorted variables that occur in the restriction of “mo” whenever it satisfies its own presupposition.

But then, is the concept of appropriate assignment unnecessary after all? For, it is likely to be the case that the fulfillment of the presupposition of “mo” automatically guarantee the existence of appropriate assignments. The answer to the question depends on whether we can find some linguistic phenomenon that needs this concept apart from “dono + N” quantification. And I think there is such a phenomenon in a semantics of Japanese interrogatives, which will be the subject of chapter 7.
Chapter 6

The interplay between “dono . . . mo” quantification and quantity phrase quantification

6.1 A problem of incorrect truth condition

We should again start with a pair of sentences, which, I am afraid, may have become too familiar by now.

(93) Dono kodomo no dono oya mo kita.
    child(ren) GEN parent(s) ∃ came
    (Any parent of any child came.)

(64) Dono kodomo no oya mo kita.
    child(ren) GEN parent(s) ∃ came
    (Any child’s parent(s) came.)

In the previous chapter, although tentatively, we came to a conclusion that they have the same presupposition. The fact that this is a tentative conclusion shows that it is not obvious that they have the same presupposition.

It is obvious, however, that they have different truth conditions. The problem with our account, which have been alluded twice before, is that according to it they would have the same truth condition, contrary to this obvious fact. Now we must remove this serious flaw in our account.

We are going to achieve this in two stages. We have been concerned with the way how an indeterminate noun phrase which appears as the restriction of
“mo” gets its semantic values, but we have said nothing about the predicate which constitutes the scope of “mo”. First, we examine the semantics of the predicate which appears as the scope of “mo” and argue that it has its own quantificational structure which is required for being the scope of “mo”. This will force us to yet again reconsider the plural quantification involved in “mo” construction, and this will lead us to the third and final version of the axiom of “mo”.

6.2 Plural definite description and adverbial quantification

Let us start with observing that there are variants of (64) with adverbial quantifiers, as the following examples show.

(64a) Dono kodomo no oya mo hitori kita.  
child(ren) GEN parent(s) ∀ one person came  
(For any child, at least one parent came.)

(64b) Dono kodomo no oya mo ryouhou kita.  
child(ren) GEN parent(s) ∀ both came  
(For any child, both parents came.)

The corresponding variants of (93), however, are ungrammatical (or, at least grammatically suspect). The situation becomes much clearer, if we consider another sentence and its variants instead of (64), which admits only a limited range of quantifiers because each child has at most two parents.

(105) Dono kodomo no tomodachi mo kita.  
child(ren) GEN friend(s) ∀ came  
(For any child, its friend(s) came.)

(106) Dono kodomo no tomodachi mo futari kita.  
child(ren) GEN friend(s) ∀ two person came  
(For any child, at least one friend came.)

(107) Dono kodomo no tomodachi mo minna kita.  
child(ren) GEN friend(s) ∀ all came  
(For any child, all of its friend came.)

(108) Dono kodomo no tomodachi mo hotondo kita.  
child(ren) GEN friend(s) ∀ most came  
(For any child, most of its friend came.)

These examples show that if the restriction α of a universal sentence “(α mo) ϕ” does not end with an indeterminate term “dono N”, the scope of the
universal quantifier “mo”, namely \( \phi \), can be adverbially modified by a quantifier. In order to see what makes such a construction possible, we had better look at how sentences like (105)–(108) are evaluated in a concrete situation.

Suppose that in the relevant context there are only three children, Taro, Hanako, and Yukiko, and that they all have friends\(^1\).

Let us start with (107). In this context, (107) is true if and only if its three instances are all true. One of them is the following, and it must be obvious what the other two are.

\[
(109) \text{Taro no tomodachi ga minnna kita.} \\
\text{GEN friend(s) NOM all came} \\
\text{(All of Taro’s friends came.)}
\]

It is clear that the occurrence of “Taro no tomodachi” in (109) is that of a definite noun phrase, because “minnna” (all) is a proportional quantifier and it demands the occurrence of a definite noun phrase, to be specific, that of a plural definite description.

Exactly the same thing can be said with (108). Its instances like

\[
(110) \text{Taro no tomodachi ga hotondo kita.} \\
\text{GEN friend(s) NOM most came} \\
\text{(Most of Taro’s friends came.)}
\]

have plural definite descriptions as their subjects.

It might be thought, however, that the situation is different with (106), which contains a non-proportional quantifier “futari” (two person). Consider one of its instances.

\[
(111) \text{Taro no tomodachi ga futari kita.} \\
\text{GEN friend(s) NOM two person came} \\
\text{(One of Taro’s friends came.)}
\]

Isn’t its subject noun phrase “Taro no tomodachi” an indefinite noun phrase? Haven’t we remarked in §3.1.3 that a non-proportional quantifier can be applied adverbially only to an indefinite noun phase?

But, we should be careful here. We should not consider (111) in isolation. (111) is presented here as an instance of a universally quantified sentence (106). When we evaluate (106), we consider in turn whether the predicate “futari kita” applies to what are referred to by the three noun phrases “Taro no tomodachi”, “Hanako no tomodachi” and “Yukiko no tomodachi”. Each of them refers to a certain definite group of people. They are definite noun phrases, just as

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\(^1\) What happens when some of them have no friends? We’ll get to this question in due course.
English noun phrases “Taro’s friends”, “Hanako’s friends” and “Jiro’s friends” are definite noun phrases, and not indefinite noun phrases.

In other words, if (111) is intended as an instance of (106), it had better be rephrased using a topic marker “wa” instead of a nominal case particle “ga”, so that the definiteness of the noun phrase becomes clear.

(112) Taro no tomodachi wa futari kita.

(Two of Taro’s friends came.)

What should be a “logical form” of this sentence with a definite noun phrase? We can utilize a notation for plural definite description, which we introduced in §3.2. If we simply combine the subject noun phrase “Taro no tomodachi” expressed in this notation with the predicate “futari kita” in order to get a formal representation of (112), then we will get this.

(113) ∃X[△(X, “Taro no tomodachi”) ∧ [Val(X, “futari”) ∧ Val(X, “kita”)]]

It is apparent, however, that this cannot be right. For, this implies that the predicate “futari kita” (two person - came) is true of Taro’s friends as a whole, and it implies, in turn, that Taro has just two friends. A correct representation is this.


It is worth remarking that there is another way of expressing what (112) says and that it is by using a construction “NP no uchi no Q” (Q among NP).2

(115) Taro no tomodachi no uchi no futari

(Two of Taro’s friends came.)

In general, a sentence of the form “NP wa Q VP” has the same truth condition as “NP no uchi no Q ga VP”, just as (112) has the same truth condition as (115). In the construction “NP no uchi no Q”, NP should be definite, and unlike “NP no Q” Q may be a non-proportional quantifier as well as a proportional one. Moreover, when Q is a non-proportional quantifier, it may be of any of the three types, namely, it may be monotone increasing like “sukunaku-tomo san-nin” (at least three persons), monotone decreasing like “ooku-tomo san-nin”

2 It might be interesting to note that if you read “η” as “no uchi no” in (114) you will get almost the same sentence as (115).
(at most three persons), or non-monotone like “choudo san-nin” (exactly three persons)\(^3\).

As we saw a monotone increasing case in (112), let us consider a monotone decreasing case (116) and a non-monotone case (117).

\[(116)\]  
\[\text{Taro no tomodachi wa ooku-tomo san-nin kita.}\]

(At most three of Taro’s friends came.)

\[(117)\]  
\[\text{Taro no tomodachi wa choudo san-nin kita.}\]

(Exactly three of Taro’s friends came.)

It is not difficult to adopt our axiom for non-proportional quantifiers to these cases. (118) gives the truth condition of (116), and (119) gives that of (118).

\[(118)\]  
\[\exists X[\triangle(X, \text{“Taro no tomodachi”}) \land \forall Y[[Y \eta X \land \text{Val}(Y, \text{“kita”})] \rightarrow \neg \text{Val}(Y, \text{“sukunaku-tomo yo-nin”})]]\]

\[(119)\]  
\[\exists X[\triangle(X, \text{“Taro no tomodachi”}) \land \exists Y[Y \eta X \land [\text{Val}(Y, \text{“choudo san-nin”}) \land \forall Z[[Z \eta X \land \text{Val}(Z, \text{“kita”})] \rightarrow Z \eta Y]]]\]

Let me give a case of a proportional quantifier as well. The following (120) has a proportional quantifier “minna” (everybody), and (121) is the representation of its truth condition in our metalanguage.

\[(120)\]  
\[\text{Taro no tomodachi wa futari ga kita.}\]

(Two of Taro’s friends came.)

\[(121)\]  
\[\exists X[\triangle(X, \text{“Taro no tomodachi”}) \land \forall Y[[Y \eta X \land \text{Val}(Y, \text{“futari”})] \rightarrow Y \eta X]]\]

Just as with a sentences of the form “NP wa Q ga VP” or “NP no uchi no Q ga VP”, Q may be either non-proportional or proportional. One noticeable feature of this construction is that Q can be accompanied by an anaphoric pronoun “sono” as the following example shows.

\[(i)\]  
\[\text{Taro no tomodachi wa sono futari ga kita.}\]

(As for Taro’s friends, two of them came.)

\[(ii)\]  
\[\text{Taro no tomodachi wa sono futari ga kita.}\]

(As for Taro’s friends, two of them came.)

\(^3\) Another way of expressing the same content is to use a sentence of the form “NP wa Q ga VP” instead of “NP wa Q VP”. Note that Q is a noun in the former, while in the latter Q is an adverbial. Compare the following with (112).

\[(i)\]  
\[\text{Taro no tomodachi wa futari ga kita.}\]

(Two of Taro’s friends came.)

\[(ii)\]  
\[\text{Taro no tomodachi wa sono futari ga kita.}\]

(As for Taro’s friends, two of them came.)

\[^4\] “sukunaku-tomo yo-nin” means “at least four persons”, which is the contrary of “ooku-tomo san-nin”. 

\[\text{90}\]
Taro no tomodachi wa minna kita.
(Taro’s friends all came.)

(120) ℧X[△(X, "Taro no tomodachi") ∧ ℧Y[△[YηX ∧ Val(Y, "kita") ∧ Val(⟨Y, X⟩, "minna")]]

This is equivalent to

℧X℧Y[△(X, "Taro no tomodachi") ∧ [△[YηX ∧ Val(Y, "kita") ∧ Val(⟨Y, X⟩, "minna")]]

which can be read as the following.

All of Taro’s friends and those among them who came are such that the latter is all of the former.

Now, we are in a position to consider (105), which I repeat here.

(105) Dono kodomo no tomodachi mo kita.
(For any child, its friend(s) came.)

Suppose as before that there are only three children, Taro, Hanako and Yukiko, and they all have friends. Then, (105) is true if and only if its three instances are all true. We get these instances by substituting “Dono kodomo” with one of the names in (105). The noun phrases like “Taro no tomodachi” in the resulting sentences should be definite noun phrases, just as it is the case with (106) and its instances. Hence, one of the instances of (105) should be (122) rather than (123).

(122) Taro no tomodachi wa kita.
(Taro’s friend(s) came.)

(123) Taro no tomodachi ga kita.
(A friend/some friends of Taro came.)

As we have been emphasizing, for (105) to be true it is not necessary that for any children all its friends came. Hence, as an instance of (105), (122) should not be read as implying that all of Taro’s friends came. Given that “Taro no tomodachi” is definite, the truth condition of (122) has to be the following.

(124) ℧X[△(X, "Taro no tomodachi") ∧ ℧Y[YηX ∧ Val(Y, "kita")]]
In other words, (122) is true if and only if some among Taro’s friends came. This fits in the pattern that is found in the sentences which have definite noun phrases as their subjects and whose predicates contain adverbial quantifiers. An important difference, however, is that in this case an adverbial quantifier does not explicitly occur.

It should be remarked, however, that when an adverbial occurs implicitly, it is not always existential. As we are going to see in §6.4, there is a class of predicates, with which the default quantification is not existential but rather universal.

6.3 “Mo” and plural definite descriptions

Hopefully our considerations about the instances of a universally quantified sentence has made it clear that plural quantification involved in “mo” is not over any group of individuals that satisfy the restriction, but the maximal groups of individuals that does so relative to a particular assignment. Our previous versions missed this important feature of “mo” quantification.

Let me explain this with an example. According to the second version of our axiom of “mo”, the truth condition of (105) is given in the following formula.

\[ (125) \forall X [\exists \sigma \text{Val}(X, \text{“dono kodomo no tomodachi”}, \sigma) \rightarrow \text{Val}(X, \text{“kita”})], \]

where “\( \sigma \)” is a meta-variable ranging over assignments for an indeterminate noun phrase “dono kodomo no tomodachi”.

Now we know that the antecedent of the conditional inside “\( \forall X \)” should be much stronger. \( X \) may not be any group of individuals which are friends of a person determined by an assignment \( \sigma \); \( X \) should be the the group consisting of all friends of this person\(^5\). As well as the condition

\[ \text{Val}(X, \text{“dono kodomo no tomodachi”}, \sigma), \]

\( X \) should also satisfy the condition that \( X \) is maximal among those satisfy the same condition, namely, \( X \) should also satisfy for the same assignment \( \sigma \) the following condition.

\[ \forall Y [Y \eta X \leftrightarrow \text{Val}(Y, \text{“dono kodomo no tomodachi”}, \sigma)] \]

It is straightforward to make a notation for a plural definite description, which we introduced in §3.2, relative to an assignment. Just let “\( \Phi(X) \)” in the definition of “\( \triangle \)” be “\( \text{Val}(X, \alpha, \sigma) \)” where \( \sigma \) is an assignment of \( \alpha \).

A notation for a plural definite description relative to an assignment

\(^5\) In this respect, the present account agrees with [Shimoyama 2006]. In particular, see its footnote 18 on p.151.
Let $\alpha$ be an indeterminate noun phrase, and $\sigma$ an assignment for $\alpha$. Then, let
\[ \triangle(X, \alpha, \sigma) \]
be an abbreviation of
\[ \text{Val}(X, \alpha, \sigma) \land \forall Y [Y \eta X \leftrightarrow \text{Val}(Y, \alpha, \sigma)] \]

Given such a notation, the antecedent in question should be written as
\[ \exists \sigma \triangle(X, \text{“dono kodomo no tomodachi”}, \sigma) \]

In general, the truth condition of a sentence of the form “(\alpha mo) \phi” should be like
\[ \forall X[\exists \sigma^\alpha \triangle(X, \alpha, \sigma) \rightarrow \ldots], \]
and not like
\[ \forall X[\exists \sigma^\alpha \text{Val}(X, \alpha, \sigma) \rightarrow \ldots], \]
as it was in the previous chapter.

This change in the semantics of “mo” quantification makes it necessary to reconsider what presupposition should be involved in “dono . . . mo” quantification. We have discussed this in the previous chapter. There we considered two options, which are the following.

(I) As “dono” turns a simple and complex noun phrases into sorted variables and the domain of a variable should be non-empty, for each noun phrase that is bound by “mo” and is associated with “dono”, there should be some individuals which are its semantic values.

(II) As the universal quantifier “mo” is a presuppositional quantifier, there is a presupposition that the indeterminate noun phrase $\alpha$ that is the restriction of “mo” is not empty, that is, there are some individuals that are the semantic values of $\alpha$.

Now we are going to revise the axiom of “mo” in such a way that a universally quantified “mo” sentence is more like a conjunction of the sentences which have plural definite descriptions as their subjects. Whether singular or plural, a definite description is regarded as a typical presuppositional quantifier. Thus, our present view seems to be in favor of (II), not (I). However, there seems to be a serious problem in thinking in this line.

Let us consider (105).

(105) Dono kodomo no tomodachi mo kita.
child(ren) GEN friend(s) \forall came
(For any child, its friend(s) came.)

As we did before in §6.2, suppose that there are only three children, Taro, Hanako, and Yukiko in the relevant domain. In this situation, (105) is equivalent to the conjunction of three sentences\(^6\), one of which is

\[
(122) \quad \text{Taro no tomodachi wa kita.}
\]

\[
\text{(Taro's friend(s) came.)}
\]

But, this time let us suppose that Taro has no friends. As “Taro no tomodachi” in (122) is a plural definite description, if a definite description is presuppositional, then (122) is neither true nor false in the supposed situation. But then, (105) itself should lack a truth value.

This means that (105) presupposes that every child in the domain has some friends. But this is a much stronger presupposition than what is required by (II), namely, there are some children in the domain who has a friend, to say nothing of (I), which requires only that there is some child in the domain. Is it reasonable to put forward such a strong condition as a presupposition of “dono . . . mo” quantification?

However natural such a reaction seems to be, it may be based on a wrong way of viewing the situation, and there is another perspective for looking at the same situation.

In our example, we started with the supposition that Taro, Hanako, and Yukiko constitute the range of “dono kodomo”. What is a justification for this?

Those of us who have a background in philosophy and approach natural language with logical tools should be very careful not to unconsciously import certain special settings in language use as a paradigm for language use in general. In the parts of language where logical tools were first applied, namely, the language of mathematical and scientific theories, the domain of quantification is fixed and does not change throughout a theory. In everyday conversation, however, the domain of quantification constantly changes as a conversation unfolds. Hence, a quantificational domain is never global as in mathematical or scientific theories; each quantification that occurs in everyday conversation has its own

\(^6\) Or, using a conjunctive “mo”, (105) is equivalent to

\[
(i) \quad \text{Taro no tomodachi mo Hanako no tomodachi mo Yukiko no tomodachi}
\]

\[
\text{GEN friend(s) and GEN friend(s) and came}
\]

\[
\text{(Taro’s friend(s), Hanako’s friend(s), and Yukiko’s friend(s) came.)}
\]

If Taro has no friends, then the plural definite description “Taro no tomodachi” refers to nobody, and (i) will be judged neither true nor false, according to the presuppositional view of a definite description.
domain that stands in a complex relation with other quantificational domains of the preceding or succeeding utterances. Such local domains are various in size, but usually they are not large and sometimes they can be very small.

Returning to our example, here is one possible view which we may take. According to it, if someone uttered (105) in the course of a conversation, the speaker intended the domain of “dono kodomo” to consist of only those children who have friends, and hence, if Taro has no friends, then he is not in the domain.

To see this is not an unreasonable view, we may note that the domain of “dono kodomo” can easily expand or contract in a single discourse. Let us consider the following series of utterances, the first of which is that of (105).

(126–a) Dono kodomo no tomodachi mo kita. (= (105))
child(ren) GEN friend(s) ∨ came

(For any child, its friend(s) came.)

(126–b) Sore de kodomo no kazu wa bai ni natta.
that with children GEN number TOP double DAT became

(With that, the number of children has doubled.)

(126–c) Dono kodomo mo okashi o moratta.
child(ren) ∀ sweets ACC received

(Each child was given some sweets.)

Anybody can see without difficulty that the domain of “dono kodomo” in (126–c) is much larger than that of the same phrase in (126–a). If the domain of a sorted variable “dono kodomo” can vary like this and we have a capacity to accommodate such a change, then it may not be unreasonable to suppose that we can understand the domain of “dono kodomo” in (105) (= (126–a)) as restricted to those children who have friends.

Thus, it is possible to defend a position on the presupposition of “mo” which is stronger than (II). As we saw in §5.4, (I) and (II) can be reformulated in terms of assignments for an indeterminate noun phrase α of the sentence “(α mo) ϕ”.

(I) A sentence of the form “(α mo) ϕ” presupposes that there is an appropriate assignment for α.

(II) A sentence of the form “(α mo) ϕ” presupposes that there exists some assignment σα such that ∃XVal(X, α, σα).

Then, the position we are now considering is this.

(III) A sentence of the form “(α mo) ϕ” presupposes that, for any assignment σα, ∃XVal(X, α, σα).
If the conjecture we proposed in §5.4 is true, then “assignment” in (II) and (III) may be replaced by “appropriate assignment”, which makes it clear that the presuppositional condition becomes stronger as we go from (I) to (III)\(^7\).

As is usually the case with presuppositional phenomena, the most difficult problem is to decide whether a presupposition is generated by semantic properties which are specific to particular expressions or constructions, or it is the result of a pragmatic inference based on some general beliefs and strategies which people make use of in verbal communication.

Of the three positions above, however, this problem does not arise with (I), because the requirement of the non-emptiness of a domain is clearly specific to indeterminate noun phrases, which are interpreted as sorted variables in our account. Hence, for those who guard against unwittingly admitting pragmatic consideration to a semantic account, (I) is the safest position to take.

If we compare the remaining two, (III) seems to be the better choice. For, it offers an explanation why “mo” is a presuppositional quantifier, while you must accept it as a brute fact if you choose (II). Whether its nature is semantic or pragmatic, the presuppositional character of a definite description is widely recognized, and according to (III) this is the source of the presuppositionality of “mo”.

Hence, in the final version of the axiom of “mo”, I adopt (I). I also present an alternate version in which (III) is adopted just as I did before with the second version of the axiom.

### 6.4 Adverbial quantifier in the predicate part.

#### Total and partial predicates

Let us take (125) again, which was supposed to give the truth definition of (105) in the previous chapter. We already know that its predicate part, namely, “Val(\(X\), “kita”)”, cannot be right, either. The predicate part which is the scope of “mo” has a more complex structure which varies depending on the sort of a quantifier that occurs adverbially in it. (105) is the simplest case, yet its predicate part contains an implicit existential quantification, namely,

\[
\exists Y [Y \eta X \wedge \text{Val}(Y, \text{“kita”).}]
\]

Thus, (105) as a whole gets the following representation in our metalanguage.

\[
(127) \forall X [\exists \sigma \triangle (X, \text{“dono kodomo no tomodachi”, } \sigma)] \rightarrow \exists Y [Y \eta X \wedge \text{Val}(Y, \text{“kita”).}]
\]

\(^7\) There is another position which is even weaker than (I), namely,

(0) A sentence of the form “(\(\alpha \text{ mo} \) \(\phi\)” presupposes that there is an assignment for \(\alpha\).

This requires only that an indeterminate term has a non-empty domain.
For other cases, we have to take account of the kind of a quantifier that occurs in the predicate part as it is the case with (106)–(108). If the predicate of a universally quantified sentence contains an adverbial occurrence of a quantifier, then depending on whether it is proportional or non-proportional, and if non-proportional, whether it is monotone-increasing, monotone-decreasing, or non-monotone, the way the predicate part contributes to the truth condition of the whole is different and we have to give a different clause for each case in our axiom of “mo”.

It is not true, however, that any quantifier that occurs adverbially in the predicate part is relevant for classifying the different cases in stating the axiom. To see that, let us consider the following sentence.

\[(128)\] Dono kodomo no oya mo tomodachi
which child GEN parent(s) mo friend(s)
o futari tsurete kita
ACC two persons bring came

(Every child’s parent(s) brought two friends of his/her/their own.)

In this sentence, a quantifier “futari” (two persons) occurs adverbially in the predicate of a sentence universally quantified by “mo”. But, it modifies “tomodachi”, which is one of the constituents of the predicate, and it is not related to the subject of the sentence. In this respect, (128) is crucially different from (106)–(108). In the latter, a quantifier that adverbially occurs in the predicate relates to the semantic values of the subject part. For example, “futari” (two person) in (106) is supposed to apply to “dono kodomo no tomodachi” (any child’s friends). In contrast, “futari” in (128) does not modify “dono kodomo no oya” (any child’s parents), which constitutes the subject part.

In stating the axiom of “mo”, we wish to classify the cases according to the adverbial quantifier \(Q\) that occurs in the predicate part \(\phi\) of the sentence of the form “(\(\alpha\) mo \(\phi\))”. The above consideration shows that we cannot do this just by looking at any quantifier that occurs adverbially in the predicate.

In order to avoid this problem, we make a simplifying assumption here.

**Assumption**

Suppose that a sentence of the form “(\(\alpha\) mo \(\phi\))” is given and that \(\phi\) contains an adverbial occurrence of a quantifier \(Q\). Then,

1. if \(Q\) modifies \(\alpha\), then \(Q\) is located at the head of \(\phi\), and
2. if \(Q\) is located at the head of \(\phi\), then \(Q\) modifies \(\alpha\).

For each of 1 and 2, there exist counterexamples. In (129), the quantifier “futari” (two persons) is not at the head of the predicate part, but it modifies “dono kodomo no tomodachi”. Thus, it is a counterexample to 1.
(129) Dono kodomo no tomodachi mo kouen ni futari kita.

(For every child, two of its friends came to the park.)

On the other hand, in the following (130), the quantifier “ni-satsu” (two copies) is at the head of the predicate part, but it clearly modifies “hon” (books).

(130) Dono kodomo no tomodachi mo ni-satsu hon o katta.

(For every child, its friend(s) bought two books.)

However, for each of them, it is rather easy to find another sentence which has the same truth condition and conforms to our assumption. For (129), it is enough to exchange “kouen ni” and “futari”, and similarly, if we move “ni-satsu” to the place just after “hon o” in (130), we get the desired sentence. Hence, I suppose that our simplifying assumption will not reduce the generality of our account too much.

Then, if the predicate part $\phi$ of a sentence of the form “$(\alpha \text{ mo } \phi)$” begins with a quantity noun $Q$, we may suppose that $Q$ is a quantifier that ranges over various groups of things which are determined by the totality of appropriate assignments for $\alpha$. On the other hand, if $\phi$ does not begin with a quantity noun, then we may suppose that the quantification contained in it is implicit. Is this implicit quantification always existential as it is the case with (105)? Consider the following pair of sentences.

(131) Dono heya no mado mo aite-iru.

(The windows of each room are open.)

(132) Dono heya no mado mo shimatte-iru.

(The windows of each room are closed.)

There seems to exist a clear contrast between these two sentences. A natural interpretation of (131) is that there are some open windows for each room, while that of (132) is that each room’s windows are all closed. This means that an implicit quantification that is supposed to exist in the predicate part of (132) “shimatte-iru” (is/are closed) is not existential, but universal. Thus, (132) has the same truth condition as
“Shimatte-iru” (is/are closed) and “aite-iru” (is/are open) are what Youngeun Yoon called “total and partial predicates” ([Yoon 1996]). This pair of concepts are intended by her to cover not only count predicates like “open” and “closed” but also mass predicates like “clean” and “dirty”. Because of this, she is working within a mereological framework with plural individuals. If we may restrict our considerations to count predicates and change her original formulation so that it is suitable to a plural logical setting, then her characterization of total and partial predicates may be expressed in this way.

**Total and partial predicates** (Yoon)

If \( A \) and \( B \) are a pair of lexicalized antonyms such that

(a) if \( A(X) \) and \( Y \eta X \), then \( A(Y) \), and,

(b) if \( B(X) \) and \( X \eta Y \), then \( B(Y) \),

then \( A \) is a total predicate and \( B \) is a partial predicate\(^8\).

An illustration is this: “open” and “closed” are lexicalized antonyms, and if a certain group of windows are said to be “closed”, then any window in that group should be closed, and if a certain group of windows are said to be “open”, then any larger group of windows that contain them is also said to be “closed”.

Total and partial predicates might be event predicates like “shimeru” (close) and “akeru” (open), as well as state predicates like “shimatte-iru” (is/are closed) and “aite-iru” (is/are open). “Shusseki-suru” (attend) and “kesseki-suru” (is absent from) are another pair of total and partial predicates which are event predicates.

(134) Dono kodomo no oya mo syusseki-shita.

child GEN parent(s) \( \forall \) attended

(A parent/parents of any child attended (the meeting).)

(135) Dono kodomo no oya mo kesseki-shita.

child GEN parent(s) \( \forall \) is/are absent from

(A parent/parents of any child is/are absent from (the meeting).)

(134) will be judged true if at least one parent of each child attended the meeting, while (135) will be judged true only when no parent of any child attended the meeting. Hence, if a formal representation of (134) is like that of (105), namely (127), then (135) should represented differently. (136) and (137) formally represents (134) and (135) respectively.

\(^8\) [Yoon 1996], p.224.
Yoon notes that apart from total and partial predicates which should form a pair of lexicalized antonyms, there are predicates that do not have lexicalized antonyms and behave like total or partial predicates. Interestingly enough, event predicates\(^9\) tend to behave like partial predicates, and state predicates\(^10\) behave like total predicates. Although Yoon’s examples are all English ones, it seems the same is true with Japanese. “Kita” (came) in (105) is an event predicate, and the representation (127) of its truth condition shows that this predicate behaves like a partial predicate. In contrast, consider a sentence with a state predicate “shougakusei da” (is a schoolchild/are schoolchildren).

(138) Dono kodomo no tomodachi mo shougakusei da.

(Any child’s friends are schoolchildren.)

If one of the children has a friend who is not a schoolchild, then does that fact make (138) false? The answer seems to be yes; (138) entails that any friend of any child is a schoolchild.

These examples might suggest that we should have a distinction between predicates which are much more general than that between total and partial predicates. Yoon seems to think in this line, when she introduces the distinction between weak and strong predicates. I have some reservations about doing this, however. In order to explain them, let me discuss another distinction between predicates, which has been important in accounting plurality in natural language.

Remember that one of the main reasons for developing plural logic on one hand and singularist ontology on the other is accounting sentences like the following.

(18) Taro to Hanako ga atta.

(Taro and Hanako met together.)

Unlike the sentence such as

(139) Taro to Hanako ga waratta.

(They are episodic predicates in Yoon’s terminology. [Yoon 1996], p.230.

\(^9\) They are episodic predicates in Yoon’s terminology. [Yoon 1996], p.230.

\(^10\) “Stative predicates” by Yoon.
(18) cannot be equivalent to a conjunction of simpler sentences as we noted in Chapter 2. The difference between (18) and (139) is frequently said to consist in the fact that (18) contains a collective, or non-distributive, predicate while (139) contains a distributive predicate. McKay characterizes distributive predicates in this way\footnote{[McKay 2006], p.5.}.

Let's say that a predicate $F$ is \textit{distributive} if the following condition holds in virtue of the meaning of the predicate $F$:

Whenever some things are $F$, each one of them is $F$.

In the notation which we have been using here, the last condition can be expressed thus.

For any $X$, if $\text{Val}(X, F)$, then $\forall x [x \in X \rightarrow \text{Val}(x, F)]$.

A predicate which does not satisfy this condition is called “non-distributive”. The predicate “waratta” (laughed) is distributive, while “atsumatta” (met) is non-distributive.

The distinction between distributive and non-distributive predicates are different from the one between total and partial predicates, and hence, also from the one between strong and weak predicates. For, “aiteiru” (is/are open) and “shimatteiru” (is/are closed) are both distributive predicates, though the former is a partial predicate and the latter is a total one.

The existence of a predicate which is both distributive and partial seems to give rise to a serious difficulty, however. Take “aiteiru” (is/are open), which is distributive and partial. Suppose that there are three windows $w_1, w_2, w_3$ which are all open. Let $w_4$ be another window which is closed. Then, $w_1, w_2, w_3$ jointly satisfy the predicate “aiteiru”. As these three windows are among the four windows $w_1, w_2, w_3, w_4$ and “aiteiru” is a partial predicate, these four windows also should satisfy the predicate “aiteiru”, according to the above characterization of a partial predicate. But, “aiteiru” is a distributive predicate as well and it means that each of $w_1, w_2, w_3, w_4$ also satisfies “aiteiru”. From this it follows that the window $w_4$ is open, contrary to the assumption. This is a contradiction.

What went wrong? The source of the trouble is, I believe, that whether a predicate is distributive or non-distributive is much more intrinsic property of the predicate than whether it is total or partial (strong or weak). A property $A$ of an expression $E$ is more intrinsic than a property $B$ of the same expression, when $E$ presents $A$ in a wider variety of environments than $E$ presents $B$. I claim that distributivity or non-distributivity of a predicate is a property that may be seen in a wide variety of environments, but that a predicate reveals its character as a total predicate or a partial one only in a limited class of environments.
These environments are at least two. One is that in which it appears as a predicate with a plural definite description with its subject, and another is that in which it appears as a scope of a universal quantifier “mo”.

Suppose $F$ is an atomic one-place predicate\textsuperscript{12}. Then these environments can be schematically expressed thus.

\begin{enumerate}
\item $(\alpha \text{ wa})F$, where $\alpha$ is a plural definite description.
\item $(\alpha \text{ mo})F$, where $\alpha$ is an indeterminate noun phrase.
\end{enumerate}

In these environments, although the predicate part consists of just $F$, in reality it has a hidden quantificational structure. In contrast, if $F$ is predicated of a proper name or a conjunction of proper names as in (18) and (139), $F$ functions just as it appears to be, namely, as a (plural) predicate. The same thing can be said with the case where $F$ is predicated of an indefinite noun phrase. Compare the following two sentences.

(140) Kono heya no mado ga shimatte-iru.
\begin{center}
this room GEN window(s) NOM closed-is/are
\end{center}
(A window / some windows of this room is / are closed.)

(141) Kono heya no mado wa shimatte-iru.
\begin{center}
this room GEN window(s) TOP closed-is/are
\end{center}
(This room’s window(s) is / are closed.)

Let us imagine what sort of situations are appropriate for uttering (140) and (141) respectively. (140) would be appropriate when you want to make sure that all the windows of your house are open and are checking every room for a closed window. If you find any closed window in any room, then you would utter (140). In contrast, (141) would be appropriate when you want to make sure that all the windows of your house are closed and are checking every room for an open window. In this situation, if you check a room and find out there is no open window in it, then you would utter (141). In the former situation, you are checking whether there is any closed window or not. In the latter, you are checking whether all the windows are closed or not.

The only difference between (140) and (141) is that the latter has a topic marker “wa” at the place where the former has a nominative particle “ga”. But this makes a big difference, because the noun phrase “kono heya no mado” now refers to all the windows of this room, and hence, it becomes a definite noun phrase. Moreover, this change from indefinite to definite forces a change in the interpretation of the predicate part of each sentence. As “kono heya no mado” in (140) is indefinite, (140) is true if there are some windows of this room which are closed. In contrast, when the same noun phrase is definite as in (141), the predicate “shimatte-iru” should be predicated of what this noun phrase refers to.

\textsuperscript{12} It should not be difficult to generalize this to predicates of two place and more.
refers to, namely, all the windows of this room, and this makes it necessary to introduce quantification in order to determine what such a predication to a totality amounts to.

In general, if a predicate is applied to a definite plurality $A$, then there are different ways in applying it to $A$; a predicate may be true of only some things among $A$, or it should be true of all $A$. This constitutes the distinction between partial and total predicates, more generally, that between weak and strong predicates.

It should be emphasized that “shimatte-iru” is a distributive predicate, and this property of the predicate is present in both (140) and (141). A distributivity and non-distributivity are properties of a predicate that are retained in most of the environments that it may occur.

If we ignore a demonstrative element introduced by “kono heya”, then the truth conditions of (140) and (141) may be stated in the following way.

\[(140') \exists X[(X, \text{“kono heya no mado”}) \land \text{Val}(X, \text{“shimatte-iru”})].\]

\[(141') \exists X[\Delta(X, \text{“kono heya no mado”}) \land \forall Y[Y \eta X \rightarrow \text{Val}(Y, \text{“shimatte-iru”})]].\]

You can see that the differences between them are two: (1) whether a noun phrase “kono heya no mado” is a definite description or not, and (2) whether a predicate “shimatte-iru” is within a scope of a quantifier other than the outer-most existential quantifier or not.

In spite of these differences, “shimatte-iru” has not lost its character of a distributive predicate. We can make explicit the distributivity of “shimatte-iru” by deploying it for each sentence.

\[(140'' \exists X[(X, \text{“kono heya no mado”}) \land \forall x[x \eta X \rightarrow \text{Val}(x, \text{“shimatte-iru”})]].\]

\[(141'' \exists X[\Delta(X, \text{“kono heya no mado”}) \land \forall Y[Y \eta X \rightarrow \forall y[y \eta Y \rightarrow \text{Val}(y, \text{“shimatte-iru”})]].\]

You may note that the semantic values of “shimatte-iru” are distributed to those at individual level in both sentences. Just as $X$ is distributed to $x$s in (140''), $Y$ is distributed to $y$s in (141'').

The distinction between total/partial or strong/weak is a distinction which becomes apparent only in certain limited contexts which include the scope of a universal quantifier “mo” and the predicate part of a sentence with a plural definite description as its subject. In these contexts, a total (strong) predicate tends to give rise to a universal reading, while a partial (weak) predicate to an existential reading. If this limitation of the context is ignored and these distinctions are applied generally, it would result in a contradiction such as we encountered above.
6.5 The final version of the axiom of “mo”

The preceding discussion has shown clearly that there exists a striking similarity between a universally quantified sentence with “mo” and a sentence which has a plural definite description as its subject\textsuperscript{13}. This suggests that we may give a semantic account of the latter kind of sentence and utilize it in giving a semantics of the former kind.

We have already indicated how the truth condition of a sentence that has a plural definite description as its subject and an adverbial quantifier in its predicate part varies according to the kind of the quantifier that appears in the predicate part. What we should do is just to put these various cases together in the form of an axiom.

In order to achieve this in a way that brings the similarity between this type of sentence and a universally quantified sentence with “mo” into sharp relief, I would like to introduce a notation for restricted quantification in our metalanguage.

You will see that this notation will help greatly in giving semantic accounts of both a sentence with a plural definite description and a universally quantified sentence with “mo”. As the present accounts of both kinds of expressions look very complicated on the surface, I hate to think what they will become without this notation.

Restricted quantification

Let Φ and Ψ be predicates in the metalanguage which contain the free occurrence(s) of the variable “X”, and Q be a quantity phrase\textsuperscript{14}. Then, restricted quantification

\[(\text{[Q]}X : \Phi)\Psi\]

is an abbreviation of a formula that is determined by a kind of a quantifier Q, namely,

(i) if Q is monotone increasing, it is

\[\exists X[[\text{Val}(X, Q) \land \Phi(X)] \land \Psi(X)].\]

(ii) if Q is monotone decreasing, it is

\[\forall X[[\Phi(X) \land \Psi(X)] \rightarrow \neg\text{Val}(X, \overline{Q})].\]

(iii) if Q is non-monotone, it is

\[\text{[Q]}\]

\textsuperscript{13} Here “subject” is used in an extended sense; a plural definite description is a subject of a sentence, if it functions as one of the arguments of the main predicate and its case need not to be a nominative one.

\textsuperscript{14} “Q” is an expression belonging to the object language, namely, a fragment of Japanese, while “[Q]” is a metalanguage expression which may be regarded as a metalanguage quantifier corresponding to “Q”.

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∃X[(Val(X, Q) ∧ Φ(X)) ∧ Ψ(X)] ∧
∀Y[(Φ(Y) ∧ Ψ(Y)) → XηY].

(iv) if Q is proportional, it is
∃X[△(X, [Φ(X)] ∧ Ψ(X))] ∧ ∃Y[Val(⟨X, Y⟩, Q) ∧ △(Y, Φ)].

Note that a quantifier Q is a one place predicate in (i)–(iii) because it is a non-proportional quantifier, while in (iv) Q is a two place predicate. Moreover, “¬Q” is a predicate which expresses the contrary of Q.

Although a combination “NP + wa” tends to occur at the start of a sentence, there are sentences which do not conform to this general tendency as the following examples show.

(142) San-nin no sensei ga sono kumi no
three-CL GEN teacher(s) NOM that class GEN
seito wa hotondo oshieta
pupil(s) TOP most taught
(Three teachers taught most of the pupils of that class.)

(143) San-nin no sensei o sono kumi no
three-CL GEN teacher(s) ACC that class GEN
seito wa minna kiratta
pupil(s) TOP all hated
(The pupils of that class all hated three teachers.)

Hence, our semantic account of a plural definite description should follow a pattern that is found in our previous version of the axiom of “mo” in the last chapter. As before, our account ignores a predicate of more than two places, but it should not be too difficult to extend it to other cases if you do not mind a lot of work of the bookkeeping sort involved.

Now we can state our axiom of a plural definite description.\footnote{In the following axiom, a plural definite description may have a nominative case or accusative one. Unlike “mo”, a topic marker “wa” is never used with a case particle for nominative or accusative case, although it may be used with a dative case particle “ni”. Hence, we should decide which case a plural definite description has in a given sentence by some clue other than a case particle. I am not going to investigate how this is done.}.

**Axiom of a plural definite description**

Let α be a noun phrase, Q a quantity noun, and ϕ a predicate which does not start with a quantity noun.
(1) Suppose that \( \phi \) is a one-place predicate. Then \( (\alpha \ wa)(Q \ \phi) \) (Q might be empty) is a sentence. Let us call this sentence “\( S \)”. There are two cases according to whether Q is empty or not.

(1.1) If Q is empty, then either (a) \( S \), namely, \( (\alpha \ wa) \ \phi \), is true if and only if

\[
\exists X[\Delta(X, \alpha) \land \exists Y[Y \eta X \land \text{Val}(Y, \phi)]],
\]

or, (b) \( S \) is true if and only if

\[
\exists X[\Delta(X, \alpha) \land \forall Y[Y \eta X \rightarrow \text{Val}(Y, \phi)]],
\]

depending on the predicate \( \phi \). Case (a) holds when \( \phi \) is a partial, or weak predicate, and case (b) holds when \( \phi \) is a total, or strong predicate.

(1.2) If Q is not empty, then \( S \) is true if and only if

\[
\exists X[\Delta(X, \alpha) \land (\exists Y Y \eta X) \text{Val}(Y, \phi)].
\]

(2) Suppose that \( \phi \) is a two-place predicate. Then \( (\alpha \ wa)(Q \ \phi) \) (Q might be empty) is a one-place predicate. Let us call this predicate “\( P \)”; in other words, “\( P \)” abbreviates “\( (\alpha \ wa)(Q \ \phi) \)”. Again, there are two cases to be considered, but we further divide each of them into two subcases.

(2.11) Suppose Q is empty and the plural definite description \( \alpha \) has a nominative case. then (a) either

\[
\text{Val}(X, P) \leftrightarrow \exists Y[\Delta(Y, \alpha) \land \exists Y[Y \eta X \land \text{Val}(\langle X, Y \rangle, \phi)]],
\]

or (b) \( \text{Val}(X, P) \leftrightarrow \exists Y[\Delta(Y, \alpha) \land \forall Y[Y \eta X \rightarrow \text{Val}(\langle X, Y \rangle, \phi)]],
\]

depending on the predicate \( \phi \).

(2.12) Suppose Q is empty and the plural definite description \( \alpha \) has an accusative case. then (a) either

\[
\text{Val}(X, P) \leftrightarrow \exists Y[\Delta(Y, \alpha) \land \exists Y[Y \eta X \land \text{Val}(\langle Y, X \rangle, \phi)]],
\]

or (b) \( \text{Val}(X, P) \leftrightarrow \exists Y[\Delta(Y, \alpha) \land \forall Y[Y \eta X \rightarrow \text{Val}(\langle Y, X \rangle, \phi)]],
\]

depending on the predicate \( \phi \).
(2.21) Suppose $Q$ is not empty and the plural definite description $\alpha$ has a nominative case. then
\[
\text{Val}(X, P) \iff \exists Y [\triangle(Y, \alpha) \land ([Q]Y : Y \eta X) \text{Val}(\langle X, Y \rangle, \phi)].
\]

(2.22) Suppose $Q$ is not empty and the plural definite description $\alpha$ has an accusative case. then
\[
\text{Val}(X, P) \iff \exists Y [\triangle(Y, \alpha) \land ([Q]Y : Y \eta X) \text{Val}(\langle Y, X \rangle, \phi)].
\]

If we hold that a definite description has a presupposition that there are some individuals that satisfy it, we have to have an alternative version of the above axiom. It is not difficult to get such an alternative version. Take the case (1.2) as an example. In the alternative version, this should be changed to this.

(1.2) Suppose $Q$ is not empty. If $\exists X \triangle(X, \alpha)$, then $S$ is true if and only if
\[
\exists X [\triangle(X, \alpha) \land ([Q]Y : Y \eta X), \text{Val}(Y, \phi)].
\]
otherwise $S$ is neither true nor false.

If similar changes are made in all the other cases as well, we will get the alternative version of the axiom.

Now it is easy to state the third and final version of our axiom for a universally quantified sentence with “mo”. What we have to do is to copy the present axiom of a plural definite description and make some necessary changes, which are surprisingly few. They are essentially (i) putting an existential quantification over assignments for an antecedent, (ii) changing the outermost quantifier from “$\exists$” to “$\forall$” and (iii) changing the main connective from “$\land$” to “$\rightarrow$”.

**Axiom of a universal quantifier “mo”** (final version)

Let $\alpha$ be an indeterminate noun phrase, $Q$ a quantity noun, and $\phi$ a predicate which does not start with a quantity noun. Suppose further that there are assignments for $\alpha$.

(1) Suppose that $\phi$ is a one-place predicate. Then “$\langle \alpha \text{mo} \rangle (Q \phi)$” (Q might be empty) is a sentence. Let us call this sentence “$S$”. There are two cases according to whether $Q$ is empty or not.
(1.1) If Q is empty, then either (a) \( S \), namely, \( \text{"(α mo) \( \phi \)"} \), is true if and only if
\[
∀X[∃σ^α \triangle (X, α, σ^α) → ∃Y[YηX ∧ Val(Y, φ)]],
\]
or, (b) \( S \) is true if and only if
\[
∀X[∃σ^α \triangle (X, α, σ^α) → ∀Y[YηX → Val(Y, φ)]],
\]
depending on the predicate \( φ \). Case (a) holds when \( φ \) is a partial, or weak predicate, and case (b) holds when \( φ \) is a total, or strong predicate.

(1.2) If Q is not empty, then \( S \) is true if and only if
\[
∀X[∃σ^α \triangle (X, α, σ^α) → ([Q]Y : YηX)Val(Y, φ)].
\]

(2) Suppose that \( φ \) is a two-place predicate. Then \( \text{"(α mo)(Q \( \phi \))"} \) (Q might be empty) is a one-place predicate. Let us call this predicate \( \text{"P"} \); in other words, \( \text{"P"} \) abbreviates \( \text{"(α mo)(Q \( \phi \))"} \). Again, there are two cases to be considered, but we further divide each of them into two subcases.

(2.11) Suppose Q is empty and the indeterminate noun phrase \( α \) has a nominative case. then (a) either
\[
Val(X, P) ⇔ ∀Y[∃σ^α \triangle (Y, α, σ^α) → ∃Z[ZηY ∧ Val((X, Z), φ)]],
\]
or (b)
\[
Val(X, P) ⇔ ∀Y[∃σ^α \triangle (Y, α, σ^α) → ∀Z[ZηY → Val((X, Z), φ)]],
\]
depending on the predicate \( φ \).

(2.12) Suppose Q is empty and the indeterminate noun phrase \( α \) has an accusative case. then (a) either
\[
Val(X, P) ⇔ ∀Y[∃σ^α \triangle (Y, α, σ^α) → ∃Z[ZηY ∧ Val((Z, X), φ)]],
\]
or (b)
\[
Val(X, P) ⇔ ∀Y[∃σ^α \triangle (Y, α, σ^α) → ∀Z[ZηY → Val((Z, X), φ)]],
\]
depending on the predicate \( φ \).

(2.21) Suppose Q is not empty and the indeterminate noun phrase \( α \) has a nominative case. then
\[
Val(X, P) ⇔ ∀Y[∃σ^α \triangle (Y, α, σ^α) → ([Q]Z : ZηY)Val((X, Z), φ)]
\]
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(2.22) Suppose $Q$ is not empty and the indeterminate noun phrase $\alpha$ has an accusative case, then

$$\text{Val}(X, P) \leftrightarrow \forall Y[\exists \sigma^\alpha \triangle(Y, \alpha, \sigma^\alpha) \rightarrow ([Q][Z : Z\eta Y]\text{Val}((Z, X), \phi)].$$

As “mo” may be used with a nominative case marker “ga” and an accusative “o”, $S$ in (1.1) and (1.2) might be “($\alpha$ mo ga)$\phi(Q)$” or “($\alpha$ o mo)$\phi(Q)$” as well. In the same way, $P$ might be “($\alpha$ mo ga)$\phi(Q)$” in (2.11) and (2.21), just as it might be “($\alpha$ o mo)$\phi(Q)$” in (2.12) and (2.22).

Just as it was the case with the axiom of plural definite description, there might be an alternative version of this axiom of “mo”. We have considered two such alternative versions in §6.3. There we argued that an alternative that we called (III) has a better choice if we think that presupposition involved is part of semantics of “mo”. Hence, I indicate how this alternative version can be gotten from the above axiom. Again it is enough to explain what changes are necessary with just one case, because it is easy to make necessary changes to other cases.

Then, take the case (2.21) this time. It should be changed to this.

(2.21) Suppose $Q$ is not empty. If the indeterminate noun phrase $\alpha$ has a nominative case, and

$$\forall \sigma^\alpha \exists Y \triangle(Y, \alpha, \sigma^\alpha),$$

then

$$\text{Val}(X, P) \leftrightarrow \forall Y[\exists \sigma^\alpha \triangle(Y, \alpha, \sigma^\alpha) \rightarrow ([Q][Z : Z\eta Y]\text{Val}((Z, X), \phi)],$$

otherwise $P$ does not have semantic values.

Let us go back to our final version (not alternative one) of the axiom of “mo” and try it on some examples. Our first example is this.

(144) San-nin ika no sensei ga dono seito mo oshieta
three-CL ≤ GEN teacher(s) NOM pupil∀ oshieta
(Less than four teachers taught every pupil.)

This sentence is a nice example for at least two reasons. First, it has both quantity phrase quantification and “dono . . . mo” quantification. Secondly, it offers an opportunity to see the working of a simple universal quantification with “mo” which has only a simple indeterminate phrase as its restriction.

We suppose that (144) is formally represented in this way.
\(((\text{san-nin ika}) \text{ no } (\text{sensei})) \text{ ga} \) \(((\text{dono seito}) \text{ mo}) \text{ oshieta}\)

This sentence is true if and only if

1. \(\forall X [\text{Val}(X, "\text{sensei}") \land \text{Val}(X, "((\text{dono seito}) \text{ mo}) \text{ oshieta}")]\)
   \(\rightarrow \lnot \text{Val}(X, "(\text{yo-nin ijyo})")\],

because "("yo-nin ijyo")" (more than three) is the contrary of "("san-nin ika")" (less than four).

Let us unpack

2. \(\text{Val}(X, "((\text{dono seito}) \text{ mo}) \text{ oshieta}")),

following the present axiom of "mo". As "dono seito mo" in (144) is in accusative and (144) does not contain an adverbial quantifier, the case (2.12) applies to 2. Moreover, as "oshieta" (taught) is an event predicate, (a) should be the right choice\(^{16}\). Hence, if "\(\alpha\)" stands for an indeterminate phrase "dono seito", then 2 is equivalent to

3. \(\forall Y [\exists \sigma \Delta(Y, "\text{dono seito}", \sigma) \rightarrow \exists Z[Z\eta Y \land \text{Val}((Z, X), "\text{oshieta}")]\].

The antecedent of the universally quantified conditional 3 is equivalent to

4. \(\exists \sigma \lbrack \text{Val}(Y, "\text{dono seito}", \sigma) \land \forall Z[Z\eta Y \leftrightarrow \text{Val}(Z, "\text{dono seito}", \sigma)]\].

And this, in turn, is equivalent to

5. \(\exists \sigma \lbrack Y \equiv \sigma("\text{seito}", 1) \land \forall Z[Z\eta \sigma("\text{seito}", 1) \leftrightarrow Z \equiv \sigma("\text{seito}", 1)]\].

The second conjunct inside the existential quantifier is logically true, because for any \(Z\) and any individual \(x\)

\[Z \equiv x \leftrightarrow Z\eta x.\]

Hence, 5 is equivalent to

6. \(\exists \sigma \lbrack Y \equiv \sigma("\text{seito}", 1).\)

In the same way as we did in §5.3, it can be shown that 6 is equivalent to

7. \(\text{Val}(y, "\text{seito}").\)

Hence 3 is equivalent to

\(^{16}\) It will turn out, however, the choice between (a) and (b) does not make any difference in this case, in which the restriction of "mo" is a simple indeterminate phrase.
8. \( \forall y [\text{Val}(y, \textit{seito}) \rightarrow \exists Z [Z \eta y \wedge \text{Val}(\langle Z, X \rangle, \textit{oshieta})]] \),

which is equivalent to

9. \( \forall y [\text{Val}(y, \textit{seito}) \rightarrow \text{Val}(\langle y, X \rangle, \textit{oshieta})] \),

because \( Z \eta y \) is logically equivalent to \( Z = y \) as we noted above.

Putting this back to 1, we get the following as the truth condition of (144).

10. \( \forall X [\text{Val}(X, \textit{sensei}) \wedge \forall y [\text{Val}(y, \textit{seito}) \rightarrow \text{Val}(\langle y, X \rangle, \textit{oshieta})] \rightarrow \neg \text{Val}(X, \textit{yo-nin ijyo})] \),

With lexical axioms, we can read this as "The teachers who taught every pupil are not more than three". This is as it should be.

Let us try another example, which has an adverbial quantifier in its predicate part. It is (108).

\[
(108) \text{Dono kodomo no tomodachi mo hotondo kita.}
\]

(For any child, most of its friend came.)

Let this be represented formally thus.

\[
(((\text{dono kodomo}) \text{ no tomodachi} \text{ mo}) (\text{hotondo kita})
\]

This time the case (1.2) of the axiom applies, and it is true if and only if

1. \( \forall X [\exists \sigma \triangle(X, \alpha, \sigma) \rightarrow ((\text{hotondo}) Y : Y \eta X \text{ Val}(Y, \textit{kita}))] \),

provided that \( \alpha \) stands for an indeterminate noun phrase "dono kodomo no tomodachi".

First, let us see what the antecedent of the conditional inside \( \forall X \) amounts to. If we expand this according to the definition of \( \triangle \), it is

2. \( \exists \sigma [\text{Val}(X, \alpha, \sigma) \land \forall Y [Y \eta X \leftrightarrow \text{Val}(Y, \alpha, \sigma^\sigma)]] \).

As \( \text{Val}(X, \alpha, \sigma^\sigma) \) is equivalent to

3. \( \exists Y [\text{Val}(Y, \textit{dono kodomo}, \sigma^\alpha) \land \text{Val}(\langle X, Y \rangle, \textit{tomodachi})] \),

which reduces to

4. \( \text{Val}(\langle X, \sigma^\alpha(\textit{kodomo}, 1) \rangle, \textit{tomodachi}) \),

2 is equivalent to
5. $\exists \sigma \left[ \text{Val}(\langle X, \sigma(\text{"kodomo"}, 1) \rangle, \text{"tomodachi"}) \land \forall Y[\text{Y} \eta X \leftrightarrow \text{Val}(\langle Y, \sigma(\text{"kodomo"}, 1) \rangle, \text{"tomodachi"})] \right].$

It is not difficult to show that this is equivalent to

6. $\exists y \left[ \text{Val}(y, \text{"kodomo"}) \land \text{Val}(\langle X, y \rangle, \text{"tomodachi"}) \land \forall Y[\text{Y} \eta X \leftrightarrow \text{Val}(\langle Y, y \rangle, \text{"tomodachi"})] \right].$

Or, using "△" we can rewrite this as

7. $\exists y \left[ \text{Val}(y, \text{"kodomo"}) \land \triangle(X, \text{Val}(\langle X, y \rangle, \text{"tomodachi"})) \right].$

What this says is that $X$ constitutes the friends of some child $y$.

As we have seen what the antecedent of 1 amounts to, let us turn to the consequent part. This is

8. ($\text{hotondo} \ Y : Y \eta X \ \text{Val}(Y, \text{"kita"})$).

As “hotondo” (most) is a proportional quantifier, 8 is equivalent to

9. $\exists Y[\exists Z[\triangle(Z, Z \eta X) \land \text{Val}(\langle Y, X \rangle, \text{"hotondo"}) \land \triangle(Y, [Y \eta X \land \text{Val}(Y, \text{"kita"})])].$

We remark that

$\triangle(Z, Z \eta X) \equiv X$

because the maximal group $Z$ such that $Z \eta X$ is $X$ itself. Hence, 9 can be simplified to

10. $\exists Y[\text{Val}(\langle Y, X \rangle, \text{"hotondo"}) \land \triangle(Y, [Y \eta X \land \text{Val}(Y, \text{"kita"})])].$

which says that those among $X$ who came are most of $X$.

Then, putting together the antecedent and the consequent and using some piece of logic, we have

11. $\forall y \forall X[\text{Val}(y, \text{"kodomo"}) \land \triangle(X, \text{Val}(\langle X, y \rangle, \text{"tomodachi"}) \rightarrow \exists Y[\text{Val}(\langle Y, X \rangle, \text{"hotondo"}) \land \triangle(Y, [Y \eta X \land \text{Val}(Y, \text{"kita"})])].$

Thus, we can conclude that (108) is true if and only if, for any child $y$ and $X$ which constitutes the friends of $y$, most of $X$ came.

It should be noted that our remark about the equivalence between 8 and 10 applies generally to the cases in which a restricted quantification appears with a proportional quantifier. This is true with both the axiom of plural definite description and that of “mo” quantification.

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6.6 A note on “hotondo”

Let us compare the truth condition we derived for (108) with that of another sentence which is closely connected with it, namely,

(145) Kodomo no tomodachi wa hotondo kita.

(child(ren) GEN friend(s) TOP most came)

(The friends of the/a child/children mostly came.)

Let (145) be formally represented as

\[((\textit{kodomo no} \textit{tomodachi}) \textit{wa})(\textit{hotondo kita})\]

Applying the case (1.2) of the axiom of plural definite description, this is true if and only if

12. \[\exists X[\triangle(X, "(\textit{kodomo no} \textit{tomodachi})") \land \exists Y[\text{Val}(\langle Y, X \rangle, "\textit{hotondo}") \land \triangle(Y, [Y \eta X \land \text{Val}(Y, "\textit{kita})])]]].\]

In order to see the difference between this and the truth condition we derived for (108), namely, 11 of the previous section, let us suppose that all the children in the context are three, Taro, Hanako, and Jiro. Suppose further that the total numbers of each child’s friends and the numbers of those among them who came are as in the following chart, and that the three children do not have any friend in common.

<table>
<thead>
<tr>
<th></th>
<th>the number of friends</th>
<th>the number of those who came</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taro</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Hanako</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Jiro</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>total</td>
<td>19</td>
<td>15</td>
</tr>
</tbody>
</table>

I suppose that (145) would be judged true in this situation, because fifteen out of the total of nineteen friends of the children showed up. Would (108) be also judged true in this situation? According to 11 which we derived as the truth condition of (108), it should be judged false. For 11 to hold, it is necessary that for each child most of his or her friends came, but in Jiro’s case only one came though he has three friends.

There seem to exist, however, some people who judge (108) to be true in this situation. For those people, (145) and (108) have the same truth condition. Does this show that the truth condition we derived for (108) is wrong?

I suppose that the following pair of sentences will be judged to have different truth conditions even by those who do not find the difference between the truth
condition of (108) and that of (145). Of course, we suppose that “kodomo” is understood to have the same extension in both sentences.

(146) a. Dono kodomo no tomodachi mo choudo san-nin kita.  
    (For any child just three of its friends came.)

   b. Kodomo no tomodachi wa choudo san-nin kita.  
    (Just three of the friends of the children came.)

Only when the children have the same three friends and no more, (a) sentence and (b) sentence have the same truth condition, and otherwise their truth conditions must be different.

If we replace non-monotone “choudo san-nin” (just three [persons]) with monotone increasing “san-nin ijyou” (more than two [persons]) in (146), similarly the truth conditions of (a) and (b) will be judged to be different. For, although (a) sentence logically implies (b) sentence, the converse is not valid. In the case of monotone decreasing quantifier like “san-nin ika” (less than four [persons]), (a) sentence does not follow from (b) sentence, though the converse does hold. Thus, in this case also, they will have different truth conditions.

Even in the case of proportional quantifier, there are some quantifier with which the difference in truth condition between (a) and (b) is felt clear. A case in point is “choudo hanbun” (just half). Let us consider the following pair.

(147) a. Dono kodomo no tomodachi mo choudo hanbun kita.  
    (For any child just half of its friends came.)

   b. Kodomo no tomodachi wa choudo hanbun kita.  
    (Just half the children’s friends came.)

Suppose as before that “kodomo” has the same extension in both sentences. It is obvious that (b) does not imply (a); even though exactly half of the friends of the children taken altogether came, it may happen that all of their friends came for some children while none came for other children. Although it may
seem at first sight that (a) implies (b), but a little thought would reveal that it is not true. For, if there is an overlap between the friends among the children, then the total of the friends who came may be less than half of all the friends the children have.

In contrast to this, “minna” (all) and “subete” (all) express the quantifiers with which (a) sentence and (b) sentence have the same truth condition, as you can see from the following pair.

\[(\text{148})\] a. Dono kodomo no tomodachi mo minna kita.
\hspace{10mm} child GEN friend(s) \(\forall\) all came

(For any child all of its friends came.)

b. Kodomo no tomodachi wa minna kita.
\hspace{10mm} child GEN friend(s) TOP all came

(All the children’s friends came.)

It would not be difficult to see that they logically imply each other. Hence, they have the same truth condition. The sameness of their truth condition, however, does not necessarily mean that they have the same “meaning” in one sense of this term. The ways the same truth condition is calculated are different between them, and this difference may be crucial to the understanding of the sentences. If we suppose that the meaning of an expression should be a correlate of its understanding, then the meaning of the two sentences must be different.

Let us go back to the pair (108) and (144). “Hotondo” (most) is a proportional quantifier expressing quantity noun, but unlike “hanbun” (half) or “minna” (all) what it expresses is a vague quantifier in that the boundary of its application is not determinate; it is vague how many people out of, say, thirty people amount to most of these people. Hence, there is always some indeterminateness with a sentence which has “hotondo”. Yet there are some cases in which “hotondo” clearly applies and some cases in which it clearly does not. Thus, there are some cases in which the truth value of a sentence with “hotondo” can be determined without problems.

One of such cases is the situation we imagined above which involves three children Taro, Hanako and Jiro. If the way their friends turned up is as is given in the chart there, it is clear that in this situation (145) is true and (108) is false. Thus, (145) does not imply (108).

To see that the converse does not hold, we should consider a case where there is an overlap between the friends of different children. This time suppose that there are four children Taro, Hanako, Jiro and Yukiko. Suppose further that each of them has exactly four friends and that there are three persons who are the common friends of all four children. Thus, Taro has four friends A, B, C and X; Hanako’s friends are A, B, C and Y; Jiro’s are A, B, C and Z; lastly, Yukoko’s are A, B, C and W. We assume that X, Y, Z, and W are all different
persons. Now suppose that these common friends A, B and C all came, but none of X, Y, Z and W came. In this situation, (108) is true because three of the four friends of each child came, but (145) is false because out of seven people who are the friends of the children less than half came.

Hence, we can safely conclude that (145) has different truth condition from (108) and that those who think that they have the same truth condition are wrong in their judgment. There are, however, some reasons why people tend to make such wrong judgments.

As we just saw, an inference from a sentence of the form “dono — mo hotondo ...” to that of the form “— wa hotonodo ...” is not valid in general. But there are some cases where we may infer from the former sort of sentence to the latter sort. For example, in many circumstances it may be allowed to infer (150) from (149)\footnote{17}.  

\begin{align*}
(149) \text{Dono kodomo ga kaita tegami mo hotondo todoita.} \\
\text{(Most of the letters any child wrote arrived.)}
\end{align*}

\begin{align*}
(150) \text{Kodomo ga kaita tegami wa hotondo todoita.} \\
\text{(The letters the children wrote mostly arrived.)}
\end{align*}

If we suppose that there are no letters which are jointly written by a number of children, then there will be no overlap between the letters written by each child. In that case, it is very likely true that (150) is also true if (149) is true. Of course, (149) may not be true even if (150) is true, because it might happen that some children wrote a great number of letters most of which arrived whereas the rest of the children wrote only a small number of letters and only less than half of them arrived. But this does not make it invalid to infer (150) from (149). Thus, one might think that there is not a big difference between (149) and (150).

In general, if there is no overlap between \(X\)’s which constitute the range of the restriction of “mo”, it is allowed to infer “— wa hotondo ...” from “dono — mo hotondo ...”.

There may be another reason why some people miss the difference in truth condition between a “dono — mo” sentence with “hotondo” and the corresponding sentence with a plural definite description. For many quantity phrases \(Q\), it is possible to turn a sentence of the form “dono — mo \(Q\) ...” to that of the form “dono — mo \(Q\) zutsu ...” without changing its truth condition. Here is an example.

\begin{align*}
(151) \text{Dono kodomo no tomodachi mo san-nin zutsu kita.} \\
\text{(149) should be compared with (65).}
\end{align*}
(For each child, three of its friends came.)

(152) Dono kodomo no tomodachi mo hanbun zutsu kita.
    child GEN friend(s) ∀ half each came
    (For each child, half of its friends came.)

These sentences have the same truth condition as those sentences which do not have “zutsu”. This word has the effect of making clear that an adverbial quantity phrase in the predicate part operates, as it were, distributively on the semantic values of “dono” phrase.

“Hotondo” (most) like “minna” (all) and “subete” (all), however, cannot be used with “zutsu”\(^{18}\). Thu unavailability of this form may help us to explain how the difference between a “dono — mo” sentence and “— wa” sentence is easily missed when they have “hotondo” in its predicate part. (In the case of “minna” and “subete” there is no difference in truth condition between these sentences.)

In sum, there are two circumstances that a sentence like (108) is mistakenly thought to have the same or almost same truth condition of a sentence like (145). First, in some cases, it is not wrong to infer the latter kind of sentence from the former kind. Secondly, while there is a device to emphasize the difference between a “dono — mo Q . . .” sentence with a “— wa Q . . .” sentence in the case of a quantity phrase like “san-nin” (three persons) and “hanbun” (half), this is not available to a quantity phrase “hotondo”, and this may lead to assimilate the two kinds of sentences.

\(^{18}\) To be sure, a similar effect can be had even with “hotondo” if we use a different construction.

(i) Dono kodomo ni tsuite mo hotondo no tomodachi ga kita.
    child in regard to ∀ most GEN friends NOM came
    (For each child, most of its friends came.)

In this sentence, a universal quantifier “mo” is applied to an adverbial phrase “dono kodomo ni tsuite” (in regard to any child) and “hotondo” is now not an adverbial but a noun which modifies a common noun.
Chapter 7

Indeterminate phrases and interrogative sentences

7.1 What is a question?

I assume that an interrogative sentence like (153) expresses a question ("POL" indicates a word-ending for a polite form¹).

(153) Dono sensei ga dono kodomo o oshie-mashi-ta ka.  
which teacher NOM which child ACC teach-POL-PAST ?  
(Which teacher taught which child?)

In the conception adopted here, a question is something external to a particular language; one and the same question can be expressed in different languages.

¹ According to [Masuoka and Takubo 1993], a Japanese interrogative sentence with indeterminate phrases should be in polite form, otherwise it is ungrammatical (p.137). Another interrogative particle "no" can be used with a non-polite form.

   (i) Dono sensei ga dono kodomo o oshieta no.  
       which teacher NOM which child ACC taught ?  
       (Which teacher taught which child?)

However, "no" cannot form an indirect interrogative.

   (ii) Dare ga kita ka shira-nai.  
       who NOM came ? know-not  
   (I don’t know who came.)

Hence, I am going to mostly discuss “ka”.

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A question like the one which is expressed by an interrogative sentence with an indeterminate term “dono + N” has a characteristics that is not always shared by other questions. It is that the range of possible answers to such a question is clearly determined by the question itself. Once you know which teacher and which child are talked about in (153), you can easily tell what are the possible answers to the question expressed by (153), though you may not know the correct or true answer. A question, however, is not always like this. There are many questions whose possible answers are not clear in advance. If you replace “dono + N” with “donna + N” in (153), the result will express a question whose possible answers cannot be foreseen in advance.

(154) Donna sensei ga donnna kodomo o oshie-mashi-ta ka.

(What sort of a teacher taught what sort of a child?)

There may be all sorts of answers to this. For example, a Japanese counterpart of any of the following can be a possible answer to it.

(a) A teacher fresh from the school taught a very bright child.
(b) The teacher is Hanako’s friend and the children are her daughters.
(c) The teacher always came late and the children were quarrelling all the time.

You can see there will be no limit to a variety of possible answers. Moreover, they are not mutually exclusive answers; in some cases, you can say any of the combinations of (a)–(c) as an answer to (154).

In contrast to this, there is a clearly defined range of possible answers to a question asked by an interrogative sentence like (153) and there is one among them which gives the true and complete answer to it, just as it is the case with a yes-no question like the one expressed by the following.

(155) Hanako wa kodomo o oshiemashita ka.

(Did Hanko taught a child (children)?)

Only one of the two possible answers to (155) must be a true one, namely, either

---

(156) Hai, oshie-mashi-ta.
     yes teach-POL-PAST
     (Yes, she did.)

or

(157) Iie, oshie-masendeshi-ta.
     no teach-POL(neg)-PAST
     (No, she did not.)

As these questions present a definite range of possible answers, we can expect their semantics will be much simpler than that of the questions like (154). And, in fact, there already exist a number of well-developed accounts for their semantics. Here I adopt an account according to which a question is a certain abstract object, which is called a “partition”\(^4\).

Suppose that there are only two teachers, Hanako and Taro, and two children, Mary and John, in the circumstance in which (153) is uttered. We can classify the totality of possible states of the world according to how they are related to each other in terms of the relation mentioned in (153). We can get such a classification by enumerating possible combinations between two teachers and two children. This can be done by considering the four possibilities, namely,

2. Hanako taught Mary.
3. Taro taught John.
4. Taro taught Mary.

and enumerate the different ways that each of these four possibilities obtains or not.

If the four possibilities are independent from each other, then there are \(2^4\) different ways of their obtaining and non-obtaining. Each of these ways constitutes what is called a “cell”. Each cell corresponds to a possible situation. If “+A” means the obtaining of a possibility A and “– A” its non-obtaining, then the following represents three different cells.

\[
\begin{align*}
(a) & \quad +1 \quad -2 \quad +3 \quad -4 \\
(b) & \quad +1 \quad +2 \quad -3 \quad -4 \\
(c) & \quad +1 \quad +2 \quad +3 \quad +4
\end{align*}
\]

(a) represents a possible situation in which Hanako and Taro both taught John, but Mary was not taught by any teacher; (b) represents the one in which Hanako

\(^3\) “POL(neg)” indicates a negative polite form.

\(^4\) See among others [Higginbotham 1993] and [Groenendijk and Stokhof 1989].
taught John and Mary both, but Taro did not teach any child; (c) represents
the one in which Hanako and Taro both taught John and Mary both.

You may see that the cells are mutually exclusive, because if two cells are
different then there must exist one among the four possibilities 1–4 which obtains
in one cell but does not obtain in the other. The cells are also jointly exhaustive
in the sense that every possible combination of obtaining or non-obtaining of the
four possibilities is found in them. We call the totality of the cells a "partition". A
partition is a "space of possibilities" ([Higginbotham 1996], p.371). This is
what a question is.

Thus, an interrogative sentence (153) expresses a question which is a par-
tition consisting of a number of cells that depends on the numbers of teachers
and children in the context. The same question is also expressed by an English
translation of (153),

(158) Which teacher taught which child?

An answer to a question is something which has the effect of narrowing down
the space of possibilities which is presented by the question. Hence, it must be
something which is incompatible with one or more cells of the partition. If
it excludes every cell of the partition but one, then it is a complete answer.
Otherwise, it is a partial answer. (159) expresses a complete answer to (153)
and (158), and (160) a partial answer to them.

(159) Hanako taught John and Mary both, and Taro did not taught
either of them.

(160) John and Mary were taught by the same teacher.

Although a complete answer is not necessarily a true answer, there must be
one among them. A true complete answer specifies a cell which corresponds
to what obtains in reality. A true answer might be partial as well. A true
partial answer specifies a range of cells among which there exits the cell that
corresponds to the reality.

Suppose that we get (161) or (162) as an answer to (153) or (158).

(161) Aristotle is a great logician.

(162) A teacher who taught a child taught a child who was taught
by a teacher.

What (161) asserts is irrelevant to the question, and what (162) asserts is a tautology and gives us no information about the space of possibilities the question presents. They are called “irrelevant answers”.

What seems to be an irrelevant answer, however, may turn out to be relevant
after all. For example,

(163) Taro does not speak English.
may express a partial answer to (153) or (158), when we believe that John and Mary cannot be taught by someone who does not speak English. This example shows that what a relevant answer to a question is depends on background beliefs ([Higginbotham 1993], p.199).

Finally, there is a sort of answers that challenge a presupposition of a question. In some circumstances, the only appropriate response to (153) or (158) might be something like this.

(164) There were no children.

The question expressed by (153) or (158) presupposes that there were teachers and children. If there were no teachers or there were no children, then there cannot be any space of possibilities that (153) or (158) purports to present. Hence, if this presupposition is not fulfilled, a response like (164) will be the only appropriate one.

It should be noted that (165) is a complete answer to (153) or (158).

(165) No teacher taught any child.

This has the effect of removing every cell but one from the space of possibilities, and hence, it gives a complete answer to the question. When someone answers (153) or (158) by uttering (165), she is not challenging its presupposition but is giving a straightforward answer. In other words, (153) or (158) does not presuppose that some teacher taught some child, as it has sometimes been claimed.

7.2 Assignments and the semantics of an interrogative sentence

One and the same question may be expressed in different languages. But each language has its own way of expressing a question. We are going to investigate how a question is expressed in Japanese.

Let us start with (153). How this sentence comes to express a question which is a partition as described in the previous section?

(153) has a form which other Japanese interrogative sentences have in common. It is this.

\[ S + \text{ka}, \]

where “\text{ka}” is what is sometimes called an interrogative terminal particle, and “\text{S}” represents an indicative sentence or a sentence-like expression in indicative mood that has indeterminate phrases. Of course, an interrogative sentence is also a sentence. But under the name “sentential expression” we understand only an indicative sentence or a sentence-like expression in indicative mood.
Just as we have been doing, we continue to suppose that “dono + N” is a sorted singular variable. Then, $S$ with an occurrence of “dono + N” may be regarded as an open sentence, which has its semantic value only relative to an assignment. We can construct a semantics for such $S$ with the resource we have developed for a semantics of “dono” quantification. Only little change is necessary for doing that.

First, we define the concept of assignment that is applicable to a sentence-like expression which has a free occurrence of an indeterminate term. In (153), the sentence-like expression with an indeterminate term is this.

$$((\text{dono sensei}) \text{ ga}) \ (((\text{dono kodomo}) \text{ o} \text{ oshiemashita})$$

Let us call such a sentence-like expression an “indeterminate sentence” just as we called a noun phrase with a free occurrence of an indeterminate term an “indeterminate noun phrase”. Hence, an indeterminate sentence is a sentential expression which contains a free occurrence of an indeterminate term. An indeterminate sentence is not a sentence in the full sense, and it cannot be used by itself in a linguistic exchange. It must be complemented by “ka” to form either a direct interrogative like (153) or an indirect interrogative which is part of a larger sentence.

The mere occurrence of an indeterminate term may not result in an indeterminate sentence, as the following examples show.

(166) Dono kodomo mo ki-mashi-ta ka.

(Has every child come?)

(167) Dono hon o dono kodomo mo yomi-mashi-ta ka.

(Which book has every child read?)

An occurrence of “dono kodomo” in (166) is bound by the succeeding “mo”, and hence, it is not free. As the part that precedes the interrogative particle

This English sentence is ambiguous; in one reading, it asks which is the book that is read by every child, while it asks of each child which book he or she has read. (167) has only the former meaning, and interestingly, “scrambling” (167) results in a Japanese sentence has the same meaning as the latter.

(i) Dono kodomo mo dono hon o yomi-mashi-ta ka.

(For each child, which book has he or she read?)

This is an instance of a quantified-in interrogative. See §8.2.
“ka” is a sentence in the full sense, (166) expresses a yes-no question. On the other hand, (167) contains two occurrences of an indeterminate term. The second occurrence is bound by “mo”, but the first occurrence was free before it is bound by the interrogative particle “ka” at the end of the sentence.

So, we need to define when an occurrence of an indeterminate term in an indeterminate sentence is free or not. But, what is an indeterminate sentence? It is a sentence-like expression which contains a free occurrence of an indeterminate term. Thus, characterizing a free occurrence of an indeterminate term in a sentencetial expression is the same as characterizing an indeterminate sentence. This can be done in a precise way only for an artificially circumscribed fragment of Japanese. We define the concept of an indeterminate sentence and a free occurrence of an indeterminate term in an indeterminate sentence for an extension of a fragment of Japanese we considered in §4.2, which we called “J”. Let us continue to call it “J”. We can utilize the definition of a noun phrase and indeterminate noun phrase in J (§4.2), and that of a free occurrence of an indeterminate term in an indeterminate noun phrase that we gave in §5.1.

Suppose that J has the following categories of expressions among other vocabularies.

- intransitive verb IV
- transitive verb TV
- quantifier particle q (“mo”, “ka”)
- case particle cp (“ga”, “o”)
- interrogative particle “ka”

We assume that the concepts of a noun phrase NP and an indeterminate noun phrase are defined as in §4.2. We also assume that the concept of a free occurrence of an indeterminate term in an indeterminate NP is defined as in §5.1.

**Definition: Sentences and indeterminate sentences in J**

This consists of three parts.

Part 1.

A declarative sentential expression S is an expression which has one of the following patterns (“[]” indicates that it may be omitted in some cases):

(i) $(NP \text{ ga})$ IV,
(ii) $(NP \text{ q [ga]})$ IV,
(iii) $(NP_1 \text{ cp}_1) (NP_2 \text{ cp}_2)$ TV,
(iv) $(NP_1 \text{ q [cp}_1]) (NP_2 \text{ cp}_2)$ TV,
(v) $(NP_1 \text{ cp}_1) (NP_2 \text{ q [cp}_2])$ TV,
(vi) \((\text{NP}_1 \, q_1 \, [cp_1]) \, (\text{NP}_2 \, q_2 \, [cp_2]) \, \text{TV}\),

provided that

1. An NP that directly precedes a quantifier particle \(q\) has to be an indeterminate NP. In particular, an NP that directly precedes a quantifier particle “\(ka\)” has to be an indeterminate term.
2. In patterns (iii)–(vi), two case particles \(cp_1\) and \(cp_2\) must be different if they both appear in them.
3. In patterns (iv) and (v), if \(q\) is “\(mo\)” then \(cp_1\) and \(cp_2\) come after \(q\), but if \(q\) is “\(ka\)” then they come before \(q\).

Part 2.

An occurrence \(D\) of an indeterminate term in a declarative sentential expression \(S\) is free if and only if either

1. \(S\) is of pattern (i) and \(D\) is free in NP, or
2. \(S\) is of pattern (iii) and \(D\) is either free in \(\text{NP}_1\) or free in \(\text{NP}_2\), or
3. \(S\) is of pattern (iv) and \(D\) is free in \(\text{NP}_2\), or
4. \(S\) is of pattern (v) and \(D\) is free in \(\text{NP}_1\).

Part 3.

1. A declarative sentential expression \(S\) is an indeterminate sentence if and only if \(S\) contains a free occurrence of an indeterminate term.
2. A declarative sentential expression \(S\) which is not an indeterminate sentence is a declarative sentence.
3. If \(S\) is a declarative sentential expression, then “\(S \, ka\)” is an interrogative sentence. If \(S\) contains any free occurrence of an indeterminate term, then it is no longer free in “\(S \, ka\)”. It is bound by “\(ka\)”.

As you see immediately, a Japanese fragment that is presented here is an extremely limited one. In particular, an interrogative particle “\(ka\)” can occur only once at the end of a sentence, and hence, only direct interrogatives can be formed in it. More generally, however, “\(ka\)” may appear at other places in a sentence. An example is this.

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This sentence contains two occurrences of an indeterminate term and two occurrences of the interrogative particle “ka”. The first occurrence of an indeterminate term, namely, “dono sensei” is bound by the first occurrence of “ka”, and forms an indirect interrogative “dono sensei ga kita ka”. (168) as a whole is a direct interrogative, and it itself is not an indeterminate sentence. (168) minus “ka” at its end is that. Let this indeterminate sentence be called “S₁”.

The first occurrence of an indeterminate term in S₁ is not free because there is an occurrence of “ka” after it, but the second occurrence of an indeterminate term in S₁ is free, although it is going to be bound by the second occurrence of “ka” in (168) as a whole.

One common feature of a quantifier particle “mo” and an interrogative particle “ka” is that they bind all the free occurrences of indeterminate terms in its restriction (NP of “NP mo”) or its scope (S of “S ka”) indiscriminately, as the following shows.

(169) Dono kodomo ga dono kodomo ni dashita
child NOM child DAT sent
dono tegami mo todoita.
letter(s) ∀ arrived
(Any letter any child had sent to any child has arrived.)

(170) Dono kodomo ga dono kodomo ni dashita
child NOM child DAT sent
dono tegami ga todoki- mashi-ta ka
letter NOM arrive- POL-PAST ?
(Which letter that which child had sent to which child has arrived?)

In (169), “mo” simultaneously binds not only two occurrences of “dono kodomo” but also that of “dono tegami”. Similarly, “ka” in (170) simultaneously binds two occurrences of “dono kodomo” and that of “dono tegami”.

This is different from the language of the standard logic, in which you can bind different occurrences of the same variable by a single application of a quantifier, but you can not bind the occurrences of different variables by it.

For example, consider a formula in the standard language of logic.

\[ \forall x [F(x, y) \rightarrow G(x, y)]. \]

The quantifier “\(\forall x\)” binds both of the two occurrence of the variable “\(x\)”, but it does not bind those of the variable “\(y\)”. If you want to bind “\(y\)”, you have to
put another quantifier like this.

\[ \forall y \forall x [F(x, y) \rightarrow G(x, y)] \] .

The characterization of a free occurrence of an indeterminate term in an indeterminate sentence is based on the similar concept for an indeterminate noun phrase. In the same way, we can use the definition of an assignment for an indeterminate noun phrase (§5.1) and that of an appropriate assignment for an indeterminate noun phrase (§5.4) to those of an assignment for an indeterminate sentence and an appropriate assignment for the same, only changing the references to an indeterminate noun phrase to those to an indeterminate sentence.

It is very reassuring that our semantics for an indeterminate noun phrase that we developed in order to account for quantification can be transferred with only small changes to that for interrogatives. Let us see how this is done with an example.

Let \( S_0 \) be an indeterminate sentence that is contained in (153). Further suppose that its formal representation is as displayed above. Then, an assignment \( \sigma \) for \( S_0 \) is a list of pairs 

\[ \langle \text{"dono sensei"}, x_1 \rangle, \langle \text{"dono kodomo"}, x_2 \rangle. \]

such that

\[ \text{Val}(x_1, \text{"sensei"}), \text{and} \]
\[ \text{Val}(x_2, \text{"kodomo"}). \]

If there are \( n \) teachers and \( m \) children, then there are \( n \times m \) different assignments for \( S_0 \).

A sentence has a truth value, \( t \) or \( f \), as its semantic value. In the case of an indeterminate sentence, it has a truth value only relative to an assignment. There is nothing difficult in adapting our procedure for deriving the truth condition of a sentence so that it may be extended for deriving the assignment-relative truth condition of an indeterminate sentence in simple cases\(^6\). We are going to work with a three place relation

\[ \text{Val}(t, S, \sigma^S), \]

where \( t \) is a truth value \text{true}, \( S \) an indeterminate sentence, and \( \sigma^S \) an assignment for \( S \). This is also written as

\(^6\) In the next chapter, we will see that we must work a little harder to give an account of indeterminate sentence quantification.
An indeterminate sentence $S$ is true relative to an assignment $\sigma^S$.

Suppose $\sigma$ is an assignment for $S_0$. Then, $S_0$ is true relative to $\sigma$ if and only if

1. $\exists X[\text{Val}(X, \text{“dono sensei”}, \sigma) \land \exists Y[\text{Val}(Y, \text{“dono kodomo”}, \sigma) \land \text{Val}((X,Y), \text{“oshiemashita”})]]$.

To get this, we proceed just as we calculated the semantic values of an indeterminate noun phrase, namely, we calculate the semantic values of each expression according to the semantic axioms we have presented so far, and if an expression to be evaluated contains a free occurrence of an indeterminate term, then we evaluate it relative to an assignment.

1 is equivalent to

2. $\exists x[x = \sigma(\text{“sensei”}, 1) \land \exists y[y = \sigma(\text{“kodomo”}, 2) \land \text{Val}(x,y, \text{“oshiemashita”})]]$.

Here an expression of the form $\sigma(N_k, k)$ refers to an individual which is paired by $\sigma$ with the $k$-th occurrence in $S$ of an indeterminate term of the form “dono $N_k$”, as it was defined in §5.1.

By logic, 2 is equivalent to

3. $\text{Val}(\sigma(\text{“sensei”}, 1), \sigma(\text{“kodomo”}, 2)), \text{“oshiemashita”})$.

This is true if and only if an object $a$ which is assigned by $\sigma$ to “dono sensei” taught an object $b$ which $\sigma$ assigns to “dono kodomo”. Moreover, from the definition of an assignment, $a$ should be a teacher and $b$ should be a child.

Just as we did in the previous section, suppose that there are only two teachers, Hanako and Taro, and two children, Mary and John, in the circumstance in which (153) is uttered. Then, there are four different assignments for $S_0$, and they are these.

\[
\begin{align*}
\sigma_1 & : \text{⟨“dono sensei”, Hanako⟩, ⟨“dono kodomo”, John⟩} \\
\sigma_2 & : \text{⟨“dono sensei”, Hanako⟩, ⟨“dono kodomo”, Mary⟩} \\
\sigma_3 & : \text{⟨“dono sensei”, Taro⟩, ⟨“dono kodomo”, John⟩} \\
\sigma_4 & : \text{⟨“dono sensei”, Taro⟩, ⟨“dono kodomo”, Mary⟩}
\end{align*}
\]

Given these assignments, from 3 above we may infer that

- $S_0$ is true relative to an assignment $\sigma_1$ iff Hanako taught John.
- $S_0$ is true relative to an assignment $\sigma_2$ iff Hanako taught Mary.
- $S_0$ is true relative to an assignment $\sigma_3$ iff Taro taught John.
- $S_0$ is true relative to an assignment $\sigma_4$ iff Taro taught Mary.
Relative to each assignment, $S_0$ may have either $t$ or $f$ as its semantic value. As each assignment represents a possible combination between a teacher and a child, if we specify which truth value $S_0$ has for each assignment, then we will have specified one of the possible situations between the teachers and children in terms of teaching relation. Thus, each possible situation can be represented by pairing each assignment with a truth value like this.

\[
\langle t, \sigma_1 \rangle \quad \langle t, \sigma_2 \rangle \quad \langle f, \sigma_3 \rangle \quad \langle t, \sigma_4 \rangle
\]

As we have just seen, our semantic theory provides us with the means to work out what are the necessary and sufficient condition for $S_0$ to have either $t$ or $f$ relative to each assignment. Thus, we can deduce from our semantic theory that the above represents a possible situation in which

- Hanako taught John,
- Hanako taught Mary,
- Taro did not teach John, and
- Taro taught Mary,

provided that $\sigma_1, \ldots, \sigma_4$ are as given above.

Given the same assignments, we know that there are $2^4 = 16$ such possible situations in total. They can be displayed in a table like the following.

\[
\begin{align*}
1. & \quad \langle t, \sigma_1 \rangle \quad \langle t, \sigma_2 \rangle \quad \langle t, \sigma_3 \rangle \quad \langle t, \sigma_4 \rangle \\
2. & \quad \langle t, \sigma_1 \rangle \quad \langle t, \sigma_2 \rangle \quad \langle t, \sigma_3 \rangle \quad \langle f, \sigma_4 \rangle \\
3. & \quad \langle t, \sigma_1 \rangle \quad \langle t, \sigma_2 \rangle \quad \langle f, \sigma_3 \rangle \quad \langle t, \sigma_4 \rangle \\
16. & \quad \langle f, \sigma_1 \rangle \quad \langle f, \sigma_2 \rangle \quad \langle f, \sigma_3 \rangle \quad \langle f, \sigma_4 \rangle
\end{align*}
\]

Each line is a “cell” which represents a possible situation, and the entire table is a partition which is the space of possibilities under the present assumption, namely, that there are only two teachers and two children.

In general, given $n$ teachers and $m$ children, the space of possibilities that (153) presents consists of $2^{n \times m}$ possibilities. Hence, if we wish to display these possibilities in the form of a table like the above, the table should consist of so many lines, each of which contains $n \times m$ pairs of an assignment and a truth value.

It might be thought that the space of possibilities a question presents could be infinite. An obvious place to find such cases would be mathematics. We might come up with an example like this.

(171) Dono kazu ga so-sū desu ka.
(Which number is a prime number?)

An utterance of (171), however, sounds strange, if your intention is to ask what are the prime numbers in general. If you say (171), then you will be interpreted as asking which are the prime numbers among a certain class of numbers, which may be given in the form of a list of numbers. In other words, the most natural interpretation of “kazu” (number) in (171) is that it refers to a certain contextually given, preferably finite, class of numbers.

If you wish to ask what the prime numbers are in general, a more natural way of doing so is to use an interrogative sentence with “donna + N”.

(172) Donna kazu ga so-sū desu ka.

(What sort of number is a prime number?)

It is not true that a noun N in “dono + N” always refers to a finite class of Ns while “donna + N” can refer to both finite and infinite domains. For example, “kazu” (number) in the following sentence obviously refers to an infinite domain.

(173) Dono kazu ni mo sore yori ookina kazu ga aru.

(For each number, there exists a number that is greater than it.)

Then, is it that N in “dono + N” should have a finite domain if it is used in an interrogative sentence? Even if it were so, it would be very difficult to justify such a claim as one that follows from the semantics of “dono”, because it seems likely that some pragmatic considerations will equally support it.

Hence, we had better allow an interrogative sentence with “dono + N” to present an infinite space of possibilities, and leave it a subject for further research whether there are any semantical or pragmatic grounds for restricting the spaces of possibilities to finite ones.

7.3 Cells, partitions, and dc-partitions

Now we can understand that an interrogative sentence like (166), which expresses a yes-no question, is a special case in which the number of free occurrences of simple indeterminate phrases is zero. Let me repeat (166).

(166) Dono kodomo mo ki-mashi-ta ka.

(Has every child come?)
As the part that precedes the interrogative particle “ka” is not an indeterminate sentence but a sentence, there is no need to evaluate its semantic value relative to an assignment. Then, there are only two possibilities, namely, either the sentence gets the truth value \( t \) or the other value \( f \). Thus, a yes-no question presents the space consisting of only two possibilities. In other words, a yes-no question is a partition that consists of only two cells, which may be represented as

\[
t
f
\]

If an interrogative sentence “\( S + ka \)” expresses a question other than yes-no question, then it determines a partition, which consists of cells each of which in turn consists of a number of ordered couples\(^7\) of a truth value and an assignment for \( S \). As a cell is a plurality of ordered couples and a partition is a plurality of such cells, the latter is a second-level plurality, namely, a plurality of pluralities. Such plurality of pluralities is called variously as “superplural”, “plurally plural” or “pluplural”, and it is still debated among philosophers whether such higher-level plurality should be admitted into our theoretical repertoire\(^8\).

There are two main problems at issue. First, are there superplurals in a natural language? Secondly, is the idea of superplural intelligible at all? Another question that naturally arises with the first question in our context is this: are there superplurals in a language like Japanese which is number-neutral? Although they are very interesting questions and worth investigating in detail, here I simply assume that the talk about superplural is quite intelligible and that it is needed for a satisfactory account of semantics of a natural language including Japanese. I also assume that our metalogic is at least as strong as second-level plural logic, that is, we can talk about plurality of pluralities of individuals\(^9\).

A cell can be represented as a list like the following. (After all, a list is a term referring to a plurality\(^10\).)

\[
\langle \tau, \sigma_1^S \rangle, \langle \tau, \sigma_2^S \rangle, \ldots, \langle \tau, \sigma_n^S \rangle
\]

where \( \tau \) is a truth value \( t \) or \( f \), and \( \sigma_i^S \) are assignments for \( S \).

The partition itself is a plurality of such pluralities. We may represent it as a table, or a list of lists.

---

\(^7\) As it was noted before, the concept of ordered couple is a primitive and not to be reduced to set-theoretic concepts.


\(^9\) An example of such higher-level plural logic is given in [Rayo 2006].

\(^10\) See a discussion about list in [Oliver and Smiley 2013], Ch.10.
Although the number of different assignments for \( S \) is finite in this example, our definition of a partition will cover both finite and infinite cases.

First, let us define a cell determined by an interrogative sentence “\( S + ka \)”.

**Definition: a cell determined by an interrogative sentence**

A cell \( c \) determined by an interrogative sentence “\( S + ka \)” is a plurality (a number of things) that satisfies the following conditions.

(i) \( \forall x [x \in c \rightarrow \exists \sigma^S [x = \langle t, \sigma^S \rangle \lor x = \langle f, \sigma^S \rangle]] \).

(ii) \( \forall \sigma^S [t, \sigma^S] \in c \lor \langle f, \sigma^S \rangle \in c \).

(iii) \( \forall \sigma^S [t, \sigma^S] \in c \rightarrow \neg \langle f, \sigma^S \rangle \in c \).  

(i) says that a cell is a plurality of ordered couples consisting of a truth value and an assignment for \( S \). According to (ii), a cell should contain either \( \langle t, \sigma^S \rangle \) or \( \langle f, \sigma^S \rangle \) for each assignment \( \sigma^S \) for \( S \). Finally, (iii) forbids that a cell contains both of the contradictory pairs \( \langle t, \sigma^S \rangle \) and \( \langle f, \sigma^S \rangle \)\(^{11}\).

If you just drop the reference to an assignment, then you will get the corresponding concept that applies to yes-no interrogatives.

The definition of a cell becomes this.

(i) \( \forall x [x \in c \rightarrow [x = t \lor x = f]] \).

(ii) \( t \in c \lor f \in c \).

(iii) \( t \in c \rightarrow \neg f \in c \).  

\(^{11}\) Conditions (ii) and (iii) can be combined into the following single condition.

\[ \forall \sigma^S [t, \sigma^S] \in c \leftrightarrow \neg \langle f, \sigma^S \rangle \in c. \]

\(^{12}\) Just as it was remarked in the previous footnote, (ii) and (iii) may be combined as

\[ t \in c \leftrightarrow \neg f \in c. \]
By (ii) a cell must contain $t$ or $f$, and, by (i) it may contain only them. But, by (iii) it cannot contain both. Hence, a cell for a yes-no interrogative is either $t$ or $f$.

The definition of a partition for an interrogative sentence is as follows.

**Definition: a partition for an interrogative sentence**

A partition $\Pi$ for an interrogative sentence “$S + ka$” is a plurality of all the cells determined by an interrogative sentence “$S + ka$”.

We will write a partition for an interrogative sentence “$S + ka$” as “$\Pi S$”. For any two different cells $c$ and $c'$ of the partition $\Pi S$, there must exist some assignment $\sigma^S$ such that $c$ contains $\langle t, \sigma^S \rangle$ while $c'$ contains $\langle f, \sigma^S \rangle$, or vice versa. This means that $S$ is true relative to $\sigma^S$ under the situation represented by one cell, and it is false under the one represented by the other cell. Thus, any two cells cannot represent the same situation, and in that sense, they are exclusive to each other.

Moreover, by the very definition of a partition, every cell determined by “$S + ka$” is found among $\Pi S$. This means that each possible way of distributing truth values relative to each assignment for $S$ is represented in $\Pi S$. This is why $\Pi S$ is exhaustive as well as exclusive.

I should note that what is in question here is the possibility of distributing truth values and it is not necessarily true that any way of distributing them represents a really possible situation. For an extreme case, consider an interrogative “$S + ka$” where $S$ is contradictory. Then, there exists no situation under which $S$ will get the truth value $t$ for any assignment $\sigma^S$. Still we can consider the various way of distributing truth values under each assignment.

It must be obvious that a partition for a yes-no interrogative consists of a cell $t$ and a cell $f$. If “$S + ka$” is a yes-no interrogative and $S$ is a contradiction (a tautology), then the cell $t$ ($f$) does not represent a possible situation; but a partition structure consists of two cells even in such cases.

Let me introduce some notation at this point.

We write a list consisting of lists $A_1, A_2, \ldots, A_n$ as

\[
A_1 \mid A_2 \mid \ldots \mid A_n,
\]

Although it is not grammatical in English, a better way of saying this is that a partition structure for “$S + ka$” are all the cells determined by the same interrogative sentence. “$\Pi$” is a plural definite description.
or, if we wish to save the space,

\[ A_1 \mid A_2 \mid \ldots \mid A_n. \]

As a partition is a list of lists which are called cells, a partition \( \Pi \) is written as

\[ c_1 \]
\[ c_2 \]
\[ \cdot \]
\[ \cdot \]
\[ c_n, \]

or,

\[ c_1 \mid c_2 \mid \ldots \mid c_n, \]

if it consists of a finite number of cells. For a general case including a partition consisting of an infinite number of cells, we write

\[ \mid_{i \in I} c_i, \]

where \( I \) denotes the totality of indices.

A partition for a yes-no question

\[ t \]
\[ f \]

or,

\[ t \mid f \]

is also a list of lists, namely, a list of one-membered lists.

Now we define a structure which we call “a partition with a designated cell”.

**Definition: a partition with a designated cell (dc-partition)**

Let \( \Pi^S \) be a partition for an interrogative sentence “\( S + ka \)” and \( c^* \) be a cell among \( \Pi^S \). Then, \( \langle \Pi^S, c^* \rangle \) is a partition with a designated cell for “\( S + ka \)”\( . c^* \) is called the designated cell of the partition.

We are going to abbreviate “a partition with a designated cell” as “dc-partition”.

It is obvious that, given a partition \( \Pi^S \), there are as many different dc-partitions for “\( S + ka \)” as there are different cells among \( \Pi^S \).

If “\( S + ka \)” is a yes-no interrogative, then there are only two dc-partitions for it, namely, \( \langle t \mid f, t \rangle \) and \( \langle t \mid f, f \rangle \), which we sometimes write as

\[ t^* \mid f \quad \text{and} \quad t \mid f^*. \]

Similarly, a dc-partition \( \langle c_1 \mid c_2 \mid \ldots \mid c_k \mid \ldots \mid c_n \rangle \) \( (1 \leq k \leq n) \) is also written as

\[ c_1 \mid c_2 \mid \ldots \mid c_k \mid \ldots \mid c_n. \]
7.4 The concept of true complete answer and an axiom of an interrogative sentence

Finally, we define the concept of the true complete answer to an interrogative sentence. As the true complete answer to an interrogative should inform us what obtains in the reality in the most detailed way that is possible with the linguistic resource contained in the interrogative, we may try to characterize it in the following way.

Let $\Pi$ be a partition determined by an interrogative sentence “$S + ka$”. Then, the true complete answer to “$S + ka$” is a cell $c$ of $\Pi$ such that

$$\forall \sigma^S \forall \tau[\langle \tau, \sigma^S \rangle \eta c \leftrightarrow \text{Val}(\tau, S, \sigma^S)].$$

This says that for any assignment $\sigma^S$, $\langle t, \sigma^S \rangle$ is among $c$ if and only if $S$ is true relative to $\sigma^S$, and $\langle f, \sigma^S \rangle$ is among $c$ if and only if $S$ is false relative to $\sigma^S$.

If we can assume the following principle of bivalence for $S$

$$\forall \sigma^S[\text{Val}(t, S, \sigma^S) \lor \text{Val}(f, S, \sigma^S)],$$

then we will have

$$\forall \sigma^S[\text{Val}(t, S, \sigma^S) \leftrightarrow \neg \text{Val}(f, S, \sigma^S)],$$

because the “principle of contradiction”

$$\forall \sigma^S[\neg[\text{Val}(t, S, \sigma^S) \land \text{Val}(f, S, \sigma^S)]$$

is supposed to be valid with any $S$.

Then, the above characterization of the true complete answer can be replaced with a simpler one, namely\(^{14}\)

$$\forall \sigma^S[\langle t, \sigma^S \rangle \eta c \leftrightarrow \text{Val}(t, S, \sigma^S)].$$

\(^{14}\) The original characterization can be rewritten as a universal generalization of the following conjunction

$$[\langle t, \sigma^S \rangle \eta c \leftrightarrow \text{Val}(t, S, \sigma^S)] \land [\langle f, \sigma^S \rangle \eta c \leftrightarrow \text{Val}(f, S, \sigma^S)].$$

The second conjunct can be derived from the first conjunct in the following way.

\begin{align*}
\text{Val}(f, S, \sigma^S) & \leftrightarrow \neg \text{Val}(t, S, \sigma^S) \\
& \leftrightarrow \neg \langle t, \sigma^S \rangle \eta c \\
& \leftrightarrow \langle f, \sigma^S \rangle \eta c
\end{align*}

The first line comes from the principle of bivalence. The second line follows by the first conjunct and the final line comes from (ii) of the definition of a cell.
When does the above principle of bivalence fail? It is when $S$ is neither true nor false relative to a certain assignment $\sigma^5$. There may be various ways that this happens. One case is when $S$ contains a vague expression as in (174), and another is when $S$ contains an empty name or description as in (175).

(174) Dono sensei ga hage-te imasu ka.
teacher NOM bald COP(POL) ?
(Which teacher is bald?)

(175) Barukan wa dono wakusei ni ichiban
Vulcan TOP planet the most
kyori ga chikai desu ka
distance NOM near COP ?
(Which planet is the nearest in distance to Vulcan?)

Suppose that Taro is one of the teachers (“sensei”) in the context in which (174) is uttered and that he is a borderline case of baldness, that is, it is very difficult to judge whether the sentence

(176) Taro wa hage da.
TOP bald COP

(Taro is bald.)

is true or false. According to a certain account of vagueness, (176) is neither true nor false in such a situation.

If we look at our definition of a partition for “$S + \text{ka}$”, we notice that for each assignment $\sigma$ for $S$, each cell must contain either $\langle t, \sigma \rangle$ or $\langle f, \sigma \rangle$. Suppose that we construct a partition $\Pi$ for (174) and that an assignment $\sigma_0$ assigns Taro to the occurrence of “dono sensei” in (174). Then, any cell of $\Pi$ should contain either $\langle t, \sigma_0 \rangle$ or $\langle f, \sigma_0 \rangle$, which means that (176) should be either true or false. Thus, if we judge that (176) is neither true nor false, no cell of the partition could represent our judgement.

This consideration teaches us two things. First, our way of constructing a partition for an interrogative sentence presupposes “the principle of bivalence”; the clause (ii) of our definition of a cell is just an expression of this principle. Secondly, if we wish to have a satisfactory account of an interrogative sentence which may contain vague expressions, then the present framework is essentially inadequate, and it should be revised, in particular, we should find a way of constructing a partition without presupposing the principle of bivalence, unless we hold an account of vagueness like Timothy Williamson’s, according to which (176) is either true or false though we may not be able to know which\footnote{[Williamson 1994].}.

\footnote{[Williamson 1994]}
Although vagueness is a serious problem for a semantic account of a natural language, I don’t believe that it raises an issue which is particularly connected with our main topic, namely quantification and interrogative construction by an indeterminate term. The case of empty name and description, however, is different. It is closer to our concern here. For, we have already discussed the presupposition involved in the use of plural definite descriptions and allowed that a sentence with a plural definite description may lack a semantic value (truth value) if it is used in a context where its existential presupposition is not fulfilled. Then, we should hold that an empty proper name like “Barukan” (Vulcan) that appears in (175) may cause a similar lack of semantic value in an interrogative sentence. If we don’t realize that “Barukan” is an empty name, then we might try to answer it by considering whether a sentence like the following is true or false.

(177) Barukan wa kasei ni ichiban kyori
Vulcan TOP Mars the most distance
gachikai.
NOM near

(Mars is the nearest in distance to Vulcan.)

But this is neither true nor false, and so are other seven sentences which have a name of a planet other than Mars. Suppose that we have constructed a partition II for an interrogative sentence (175). As there are eight planets in all, there are the same number of assignments $\sigma_1, \ldots, \sigma_8$ and each cell of II consists of eight couples of a truth value and an assignment. This means that it is not allowed that an indeterminate sentence “Barukan wa dono wakusei ni ichiban chikai desu” (= (175) minus the interrogative particle “ka”) may be neither true nor false relative to an assignment.

This shows again that the present concept of partition presupposes that there is no truth value gap in what is asked about. Thus, we cannot apply the concept of partition to an interrogative sentence like (175).

Is this a defect of the present approach? In this case at least, I believe, it is not so. What will be a natural reaction to (175)? Instead of trying to answer the question, you would point out that “Barukan” is a name of a non-existent planet which was once erroneously thought to exist. In other words, you would challenge the presupposition of (175) that a planet called “Barukan” exists.

The present approach excludes a question like (175) which has a false existential presupposition from getting a semantic value, and in doing so, it may help to clarify what is involved in calling a question a “wrong question”. The concept of wrong question or illegitimate question is a familiar one in philosophy; we frequently hear that some philosophical question has not received any satisfactory answer, not because it is very difficult to answer it, but because it is in reality an illegitimate question that is based on some false presupposition.
The wrongness of a question, however, may not be always caused by a false existential presupposition. Nor the failure of the principle of bivalence always lead to a wrong question. For one thing, a question with a vague expression like (174) does not seem to be a wrong or illegitimate question. Even though we may not be able to give a complete answer to (174) because we may be unsure whether a certain person is bald or not, we do not reject (174) out of hand because of that. At any rate, we may be able to give a partial answer to (174), citing some unproblematic cases of baldness and non-baldness.

It is obvious that the present account of question has many limitations, but at least it supplies us with a clear and simple picture about how some forms of questions can be semantically accounted, and it owes its clarity and simplicity to the assumption of the bivalence. This assumption also helps us to recognize and theorize about one important class of wrong questions, namely, questions with false existential presuppositions, as we will see later.

We had better record our basic assumption or policy of our account of question in the form of a principle.

**Basic assumption on a semantic value of an interrogative**

(A) If an indeterminate sentence $S$ lacks a semantic value (a truth value) relative to some assignment, then an interrogative sentence “$S + ka$” lacks a semantic value either.

We suppose that the following is part of this assumption.

(A′) If a sentence $S$ lacks a semantic value (a truth value), then an interrogative sentence “$S + ka$” lacks a semantic value either.

Thus, we assume that the concept of true complete answer is defined only for $S$ for which the principle of bivalence holds. Hence, the definition of the true complete answer to a question is this.

**Definition: the true complete answer to an interrogative sentence**

Let $\Pi$ be a partition for an interrogative sentence “$S + ka$”, and suppose that the principle of bivalence holds for $S$. Then, the true complete answer to “$S + ka$” is a cell $c$ of $\Pi$ such that

$$\forall \sigma^S[(t, \sigma^S)\eta c \leftrightarrow \text{Val}(t, S, \sigma^S)].$$

Obviously, the true complete answer to a yes-no interrogative is given by

$$t\eta c \leftrightarrow \text{Val}(t, S),$$

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in other words,
\[ t = c \leftrightarrow \text{Val}(t, S). \]
This means that \( t \) is the true complete answer when \( \text{Val}(t, X) \) and that \( f \) is the true complete answer when \( \neg \text{Val}(t, X) \).

Now we are in a position to state our semantic axiom for an interrogative sentence.

**Axiom of an interrogative sentence**

Let “\( S + \text{ka} \)” be an interrogative sentence, and suppose that the principle of bivalence holds for \( S \) or \( S \) has a semantic value for any assignment for \( S \). Then,

\[ \text{Val}(X, “S + \text{ka}”) \text{ if and only if } X \text{ is a de-partition } (\Pi^S, c^*) \text{ for “} S + \text{ka}” \text{ such that } c^* \text{ is the true complete answer to “} S + \text{ka}” \text{.} \]

Suppose that \( S \) is a sentence, and hence, “\( S + \text{ka} \)” is a yes-no interrogative. Then, according to this axiom, the semantic value of this yes-no interrogative is either a partition \( \langle t \mid f, t \rangle \) if \( S \) is true, or a partition \( \langle t \mid f, f \rangle \) if \( S \) is false.

Take a yes-no interrogative (166), which I repeat here.

(166) Dono kodomo mo ki-mashi-ta ka.
child ∀ come-POL-PAST ?
(Has every child come?)

The semantic value of (166) depends on whether an indicative sentence

(178) Dono kodomo mo ki-mashi-ta.
child ∀ come-POL-PAST
(Every child has come.)

is true or not. If (178) is true, the semantic value of (166) is \( \langle t \mid f, t \rangle \), and if (178) is false, then it is \( \langle t \mid f, f \rangle \). As our semantic axioms tell when an indicative sentence (178) is true, we know when a yes-no interrogative sentence get an answer “yes” or “no”, though we don’t know which is the right answer.

Now, suppose that \( S \) is an indeterminate sentence with the free occurrences \( D_1, D_2, \ldots, D_n \) of indeterminate terms. Let \( \text{Num}(k, C) \) \((1 \leq k \leq n)\) be the total number of the individuals that are the semantic values of \( D_k \) in the context \( C \). The total number of different assignments for \( S \) is given by

\[ \text{Num}(1, C) \times \text{Num}(2, C) \times \ldots \times \text{Num}(n, C) \]
Let this be designated by $N^{16}$. Then, the semantic value of the interrogative sentence “$S + ka$” uttered in the context $C$ is a dc-partition $(\Pi, c^*)$ such that

(i) $\Pi$ is a partition consisting of $2^N$ cells, each of which is an $N$-membered list of ordered couples of a truth value and an assignment for $S$, and

(ii) $c^*$ is the cell which is the true complete answer to “$S + ka$”.

As an example, consider the following sentence.

(179) Dono kodomo ga ki-mashi-ta ka.

child NOM come-POL-PAST ?

(Which child has come?)

This sentence has only one occurrence of an indeterminate term “dono kodomo”. Suppose that (179) is uttered in the context where there are only three children, Hanako, Taro, and Jane. Thus, there are three assignments for an indeterminate sentence “Dono kodomo ga ki-mashi-ta”. Let them be $\sigma_1, \sigma_2,$ and $\sigma_3$, and suppose that they respectively assign Hanako, Taro and Jane to the occurrence of “dono kodomo”. Then, the partition $\Pi_0$ for (179) uttered in this context is given by the following table.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>$(t, \sigma_1)$, $(t, \sigma_2)$, $(t, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_2$</td>
<td>$(t, \sigma_1)$, $(t, \sigma_2)$, $(f, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_3$</td>
<td>$(t, \sigma_1)$, $(f, \sigma_2)$, $(t, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_4$</td>
<td>$(t, \sigma_1)$, $(f, \sigma_2)$, $(f, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_5$</td>
<td>$(f, \sigma_1)$, $(t, \sigma_2)$, $(t, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_6$</td>
<td>$(f, \sigma_1)$, $(t, \sigma_2)$, $(f, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_7$</td>
<td>$(f, \sigma_1)$, $(f, \sigma_2)$, $(t, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_8$</td>
<td>$(f, \sigma_1)$, $(f, \sigma_2)$, $(f, \sigma_3)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Further suppose that Hanako and Jane have come but Taro has not come, and hence that the true complete answer to (179) in this context is $c_3$. Then, the semantic value (179) has in the context is a dc-partition $(\Pi_0, c_3)$.

Now we may consider what the semantic function of an interrogative particle “ka” is. In general, the function of “ka” is two-fold. First, it operates on

\[ \text{card}(d(D_1, C) \times d(D_2, C) \times \ldots \times d(D_n, C)), \]

which is denoted by “$\kappa$”. Then, the partition $\Pi$ will consist of $2^\kappa$ cells. The use of set-theoretic apparatus may be justified on the ground that it comes from the subject talked about in some object language, and not from our semantic theory. Our metalanguage remains the language of plural logic which itself has no commitment to the existence of sets.

---

\(^{16}\) Suppose $d(D_1, C), d(D_2, C), \ldots, d(D_n, C)$ are the domains of indeterminate terms $D_1, D_2, \ldots, D_n$ in the context. If some of these domains are infinite, then the total number of different assignments is given by
the semantic value or values of $S$ and returns a partition as one element of
the semantic value of an interrogative “$S + \text{ka}$”. Secondly, it binds the free
occurrences of indeterminate terms, if there are any. These two functions of
“ka” can be most clearly seen if a partition is displayed in the form of a table as
above. There are two directions in a table, a vertical one and a horizontal one.

The vertical direction is essential for a question to present a space of possi-
bilities. If “$S + \text{ka}$” is a yes-no interrogative, $S$ has either $t$ or $f$ as its semantic
value. Adding the interrogative particle “ka” turns either of them into the same
partition consisting of two cells $t$ and $f$, and couples it with $t$ or $f$ depending on
whether $S$ corresponds to the reality or not. If $S$ is an indeterminate sentence
with indeterminate terms, then $S$ has a semantic value only relative to an as-
signment for $S$. The actual state of affairs determines the distribution of truth
values relative to each assignment, but there are other possible situations that
correspond to other ways of distributing truth values to each assignment, and
they constitute the space of possibilities which “ka” makes out of the various se-
monic values of the indeterminate sentence $S$ relative to different assignments.

The horizontal direction is related to the power “ka” has for binding a free
occurrence of an indeterminate term in $S$. This explains why a yes-no interro-
gative does not have a horizontal dimension. Each horizontal line represents one
particular way of distributing truth values relative to each of all the assignments
for $S$, and hence, the table as a whole displays all the ways of distributing truth
values to all the assignments. Thus, it contains two-fold quantification, first for
all the assignments, and secondly for all the different ways of distributing truth
values. In this way, any free occurrence of an indeterminate term in $S$ is bound.

7.5 *Sinn* and *Bedeutung* of an interrogative sen-
tence

In his late paper “Thoughts (Der Gedanke)” (1918–19), Frege wrote the fol-
lowing about a yes-no interrogative, which he called “a propositional question
(Satzfrage)”.

An interrogative sentence and an assertoric one contain the same
thought (Gedanke); but the assertoric sentence contains something
else as well, namely assertion. The interrogative sentence contains
something more too, namely a request. Therefore two things must be
distinguished in an assertoric sentence: the content, which it has in
common with the corresponding propositional question; and asser-
tion. The former is the thought or at least contains the thought\(^{17}\).

As for what he calls a “word-question (Wortfrage)”, namely a wh-interrogative,
Frege leaves it out of consideration for the reason that it is not a complete sen-

\(^{17}\) [Frege 1984], p.355.
tence, and hence, does not express a thought. It is almost certain, however, that he allows it to have a sense (Sinn) just as a yes-no interrogative has. It is just that only a complete sentence can have a thought as its sense. A wh-interrogative can have a sense which is not a thought. Although the following consideration can be easily extended to a wh-interrogative as well, for simplicity’s sake, let us consider only a yes-no interrogative “S + ka”, where S is a sentence.

One of the basic principles of Fregean semantics is that an expression’s Sinn determines its Bedeutung. Hence, it might be thought that S and “S + ka” would have the same Bedeutung if they have the same Sinn. It is very likely, however, that he did not think an interrogative sentence had its own Bedeutung as a declarative sentence had a truth-value as its Bedeutung. There is no place in his writings which suggests otherwise.

It is well known that Frege thought that there were sentences which have a Sinn without having a Bedeutung (a truth value), for example, a sentence with an empty description. I know no case, however, in which two expressions share the same Sinn but one has a Bedeutung while another lacks it. But it does not prove by itself that such cases are impossible. Let us see whether it is so.

Let S be a declarative sentence, and S′ an interrogative sentence that corresponds to S. How is it possible that S has a Bedeutung while S′ lacks it? There are two possibilities. Either (i) S′ contains an expression which does not occur in S and lacks a Bedeutung, or (ii) S and S′ consist of exactly the same words, but a certain construction in S′ lacks a Bedeutung.

(i) A yes-no interrogative sentence in Japanese is formed from a declarative sentence by adjoining an interrogative particle “ka”. If “S + ka” lacks a Bedeutung, then the culprit must be this “ka”; “ka” would be a word without Bedeutung. As S and “S + ka” are not different in Sinn by assumption, “ka” must have no influence on the Sinn of the sentence which it occurs. But then, how the addition of “ka” can influence the Bedeutung of the resulting sentence?

(ii) In a language like German and English, an interrogative sentence may be formed from a declarative sentence by changing a word order, just as “Is Taro a student?” comes from “Taro is a student”. If the former lacks a Bedeutung but the latter has one, then its cause must be the difference in word order. There are many cases in which the change in word order makes the difference in Bedeutung of an expression. For example, if you change “John loves Mary” to “Mary loves John”, you might change a true sentence into a false one, namely, you might change the Bedeutung of the sentence. But it is rather hard to see how changing the word order robs a sentence of its Bedeutung unless it turns a sentence into a nonsense.

You might protest that this whole discussion is misguided; in the passage quoted above, Frege is saying that an interrogative sentence and an assertoric one can share a Sinn but have to be different in their forces (Kraft); the former has an interrogative force, while the latter has an assertoric force; thus, the addition of an interrogative particle or changing a word order affects only the
force of a sentence and nothing else.

Construing the difference between interrogative and declarative (assertoric) sentences as that in force, however, is hopelessly inadequate in accounting the fact that there are various interactions between them\(^{18}\). The most serious problem is that the concept of force applies only to an entire sentence and never to part of a sentence.

Frege realized full well that a declarative sentence can occur as part of another declarative sentence without having any assertoric force, for example, as an antecedent of a conditional sentence. But it seems that he did not pay sufficient attention to the similar phenomenon in the case of interrogative sentences. First, a declarative sentence can occur as part of an interrogative sentence as the following example shows.

\begin{equation}
(180) \begin{array}{ll}
\text{Taro ga kita ra Hanako wa kuru} & \text{NOM came if TOP come} \\
\text{darou ka} & \text{will ?}
\end{array}
\end{equation}

(Will Hanako come if Taro comes?)

Second, an interrogative sentence can occur as part of another interrogative sentence as well as declarative sentence. (168) above is an example of an interrogative sentence’s occurring as part of another interrogative one, and the following is an example of an interrogative sentence’s occurring in a declarative sentence.

\begin{equation}
(181) \begin{array}{ll}
\text{Dono kodomo ga kita ka Taro wa shitte-iru.} & \text{child NOM came ? TOP know} \\
\text{ka Taro wa shitte-iru.} & \text{Taro knows which child came.)}
\end{array}
\end{equation}

This sentence consists of two parts, the first part “dono kodomo ga kita ka” (which child came) is an interrogative sentence\(^{19}\).

The last example raises a serious problem for the position that tries to banish interrogative expressions from the realm of \textit{Bedeutung} and put them into that of force. As (181) is a declarative sentence, it should have a \textit{Bedeutung}, that is, (181) is either true or false in the context in which it is uttered. By the principle of compositionality of \textit{Bedeutung}, which is one of the basic principles of Fregean semantics, the \textit{Bedeutung} of (181) should be determined by the \textit{Bedeutungen} of its constituents, in particular, that of the clause “dono kodomo ga kita ka” (which child came). This means that this clause, which is an interrogative sentence, should be assigned a \textit{Bedeutung}.

\(^{18}\) This is a point Belnap has emphasized in several places. For example, see [Belnap 1990].

\(^{19}\) As its verb “kita” is not in polite form, it sounds strange if it is used by itself. Yet it is an interrogative sentence.
An interrogative sentence must have not only a *Sinn* but also a *Bedeutung*. It is not that an interrogative sentence lacks a *Bedeutung*; it has to have a *Bedeutung* which is different in kind from that of a declarative sentence.

According to our account, an interrogative sentence has both a *Bedeutung* and a *Sinn*, whether it is a yes-no interrogative or not. The concept of semantic value corresponds to Fregean *Bedeutung*. Thus, the *Bedeutung* of an interrogative sentence “$S + ka$” is a dc-partition $(\Pi^S, c^*)$ such that $c^*$ is the true complete answer to “$S + ka$”. An interrogative particle “$ka$” turns the truth value which is the *Bedeutung* of a declarative sentence $S$ to the dc-partition which is the *Bedeutung* of an interrogative sentence “$S + ka$”.

We hold with Frege that a *Sinn* of an expression is the way its semantic values are determined. Thus, the *Sinn* of an interrogative sentence “$S + ka$” is the way its semantic value, namely $(\Pi^S, c^*)$, is determined.

An account of the semantics of an interrogative sentence in terms of partitions is not uncommon in the literature. It may even be the paradigmatic account of the semantics of an interrogative sentence in formal semantics. But, the present account differs from those which are found elsewhere in at least two respects. First, what is usually taken to be the semantic value of an interrogative sentence is a partition, not a pair of a partition and the true complete answer (dc-partition). Secondly, a more common conception of a partition appeals to possible worlds and propositions, while the present account does not.

Let us begin with the question why we assign a dc-partition to an interrogative sentence as its semantic value, not a partition.

One merit of doing so is that we can assign different semantic values to an interrogative sentence “$S + ka$” and its negative counterpart “$S' + ka$” such as the following pair.

(179) Dono kodomo ga ki-mashi-ta ka.
child NOM come-POL-PAST ?

(Which child has come?)

(181) Dono kodomo ga ki- mas-en- deshi-ta ka.
child NOM come POL-NEG POL-PAST ?

(Which child has not come?)

(179) and its negative counterpart (181) determine the same partition, but the cells that are the true complete answer to them are different. If $c$ is a cell of a partition $\Pi^S$ determined by an interrogative sentence “$S + ka$”, then we can single out a particular cell which we call the contrary of $c$.

**Definition:** the contrary of a cell
c′ is the contrary of c if and only if
\[ \forall \sigma^S[(t, \sigma^S)\eta c \leftrightarrow (f, \sigma^S)\eta c']. \]

We will denote the contrary of c by \( \overline{c} \).

If \( \langle \Pi^S, c \rangle \) is the semantic value of “\( S + ka \)”, then \( \langle \Pi^S, \overline{c} \rangle \) is the semantic value of “\( \overline{S} + ka \)”. In particular, if \( S \) is a sentence, then the semantic value of a yes-no interrogative “\( S + ka \)” is \( \langle t | f, t \rangle \) or \( \langle t | f, f \rangle \), and the semantic value of the negative interrogative “\( \overline{S} + ka \)” is \( \langle t | f, f \rangle \) or \( \langle t | f, t \rangle \) respectively. For example, consider the following pair.

(182) Taro wa ki-mashi-ta ka.
   (Did Taro come?)
   TOP come-POL-Past ?

(183) Taro wa ki-mas-en-deshi-ta ka.
   (Didn’t Taro come?)
   TOP come POL-NEG POL-PAST ?

Suppose that Taro did come in the situation where (182) and (183) are asked. Then the semantic value of (182) would be \( \langle t | f, t \rangle \) and that of (183) \( \langle t | f, f \rangle \). (182) would be answered truly by

(184) Hai, ki-mashi-ta.
   Yes come-POL-PAST
   (Yes, he came.)

and (183) would be answered truly by

(185) IIe, ki-mashi-ta.
   No come-POL-PAST
   (Literally: No, he came.)

Note that (184) and (185) both assert that Taro came, even though one is preceded by “hai” and another by “iie”\(^{20} \). This fact might tempt us to think that “\( S + ka \)” and “\( \overline{S} + ka \)” have the same Bedeutung. But, it is wrong to think so. First of all, when \( S \) is not a complete sentence, namely, “\( S + ka \)” is a wh-interrogative, the answers to “\( S + ka \)” and “\( \overline{S} + ka \)” are different. Consider

\(^{20} \) Whether \( S \) is positive or negative, a Japanese speaker uses “hai” to express an agreement to \( S \) of “\( S + ka \)” and “iie” to express a disagreement. In this, Japanese “hai” and “iie” are different from English “yes” and “no”, the choice of which is determined by whether \( S \) is positive or negative. Thus, the English counterparts to (182) and (183) would be answered by the same sentence “Yes, he came” if Taro did come.
(179) and (181). Suppose that there are three children, Taro, Hanako, and Jiro, and only Jiro came. Then, the true complete answer to (179) will be

(186) Jiro ga ki-mashi-ta.
     NOM come-POL-Past
     (Jiro came.)

and, the true complete answer to (181) will be

(187) Taro to Hanako ga ki-mas-en deshi-ta.
     and NOM come POL-NEG POL-PAST
     (Taro and Hanako did not come.)

Or, using a construction “NP + igai + no” (other than NP), we may give the true complete answer to (181). For that, we do not need to know all the names of the children who did not come.

(188) Jiro igai no dono kodomo mo
come POL-NEG POL-PAST
other than which child ∀
ki-mas-en deshi-ta.
     come POL-NEG POL-PAST
     (Every child other than Jiro did not come.)

The last example shows that, for any pair of two interrogatives “S + ka” and “S + ka”, if we know the true complete answer of one, then we know that of the other. Could this be a ground for assigning the same Bedeutung to the two interrogatives?

Once we know what the true complete answer is for one of the two interrogatives, we also know the true complete answer to the other, but we should know in the first place what the true complete answer is to either of them. Frege emphasized the importance of going from grasping a thought to ascertaining its truth value.

An advance in science usually takes place in this way: first a thought is grasped, and thus may perhaps be expressed in a propositional question; after appropriate investigations, this thought is finally recognized to be true. We express acknowledgement of truth in the form of an assertoric sentence.21

For a declarative sentence S, we don’t need to know the truth value of S in order to know what thought S expresses. In other words, the knowledge of its Bedeutung is not necessary for that of its Sinn. We know that the Bedeutung

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21 [Frege 1984], p.356.
of \( S \) must be either \( t \) or \( f \), but may not know which. In general, the knowledge of its \( \text{Sinn} \) does not amount to that of its \( \text{Bedeutung} \).\(^{22}\)

It is just the same with an interrogative sentence. We don’t need to know what its true complete answer is in order to know what question it expresses. Let \( S \) be a sentence. Then “\( S + \text{ka} \)” is a yes-no interrogative. We know its \( \text{Bedeutung} \), namely its semantic value, is either \( \langle t \mid f, t \rangle \) or \( \langle t \mid f, f \rangle \), but may not know which.

To utter a declarative sentence is typically to claim that it is true. In a similar way, to utter an interrogative sentence is typically to request for its true answer. Whether it is declarative or interrogative, the semantic value of a sentence should be closely related to what uttering the sentence is supposed to accomplish. If the semantic value of an interrogative sentence were just the partition determined by it, then uttering it would only amount to present a certain space of possibilities, which is insufficient to make the essential link between an interrogative sentence and the activity of asking for an answer. This is the basic reason why I have decided to make the semantic value of an interrogative sentence a combination of a partition and one of its cells which is the true complete answer. Having different semantic values for “\( S + \text{ka} \)” and “\( \overline{S} + \text{ka} \)” is only a consequence of this basic decision.

A semantic account of an interrogative sentence in terms of partitions is most commonly given by making an appeal to possible worlds and propositions. Let me give an account for an interrogative sentence with a “\( \text{dono} + N \)” phrase in this way. As it is customary to formulate such an account in a set-theoretical framework, I will freely use set-theoretic concepts in the following.

We suppose that there is a set \( W \) of possible worlds. One of these worlds is the actual world, which is denoted by “\( @ \)”. A proposition \( P \) is a subset of \( W \). Given a sentence \( S \), the proposition expressed by \( S \) is defined by the following.

\[
\{ w \mid \text{Val}(t, S, w) \},
\]

where the semantic value relation \( \text{Val}(t, S, w) \) is relativised to a world. This relation is further relativised to an assignment, when we define the proposition expressed by an indeterminate sentence \( S \) under an assignment \( \sigma^S \), namely, it is

\[
\{ w \mid \text{Val}(t, S, w, \sigma^S) \}.
\]

The semantic value relation relativised to a world is related to the unrelativised one through the concept of the actual world, namely,

\[
\text{Val}(t, S) \iff \text{Val}(t, S, @), \text{ and,} \\
\text{Val}(t, S, \sigma^S) \iff \text{Val}(t, S, @, \sigma^S).
\]

\(^{22}\) Of course, there are a certain class of sentences which are “true in virtue of meaning”; the knowledge of their \( \text{Sinn} \) is sufficient to that of their \( \text{Bedeutung} \).
Let \( \Sigma^S \) be the set consisting of all assignments for \( S \). A bipartition of \( \Sigma^S \) is a pair \( (X, \Sigma^S - X) \). Although it may not be appropriate to call “bipartition”, \( (X, \Sigma^S - X) \) can be a bipartition even when \( X \) or \( \Sigma^S - X \) is an empty set. A cell determined by a bipartition \( (X, \Sigma^S - X) \) of \( \Sigma^S \) is a set of possible worlds defined by

\[
\bigcap_{\sigma^S \in X} \{ w | \text{Val}(t, S, w, \sigma^S) \} \cap \bigcap_{\sigma^S \in \Sigma^S - X} \{ w | \text{Val}(f, S, w, \sigma^S) \},
\]

and will be denoted by “\( c(X) \)”. Again let us take (153) as an example.

(153) Dono sensei ga dono kodomo o
which teacher NOM which child ACC
oshie-mashi-ta ka.
teach-POL-PAST ?
(Which teacher taught which child?)

Just as we have done in §7.1, let us suppose that there are only two teachers, Hanako and Taro, and two children, Mary and John in the context in which (153) is uttered. We saw there were four assignments for the indeterminate sentence “Dono sensei ga dono kodomo o oshie-mashi-ta” (let us refer to it by “\( S \)”), namely,

\[
\begin{align*}
\sigma_1 &: \text{ (“dono sensei”, Hanako), (“dono kodomo”, John)} \\
\sigma_2 &: \text{ (“dono sensei”, Hanako), (“dono kodomo”, Mary)} \\
\sigma_3 &: \text{ (“dono sensei”, Taro), (“dono kodomo”, John)} \\
\sigma_4 &: \text{ (“dono sensei”, Taro), (“dono kodomo”, Mary)}
\end{align*}
\]

There are sixteen bipartitions altogether. They can be enumerated like this.

1. \( \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}, \emptyset \)
2. \( \{\sigma_1, \sigma_2, \sigma_3\}, \{\sigma_4\} \)
3. \( \{\sigma_1, \sigma_2, \sigma_4\}, \{\sigma_3\} \)
    \[\ldots\]
15. \( \{\sigma_4\}, \{\sigma_1, \sigma_2, \sigma_3\} \)
16. \( \emptyset, \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\} \)

Each bipartition determines a cell. For example, the cell determined by the first bipartition in the above list is

\[
\begin{align*}
\{ w | \text{Val}(t, S, w, \sigma_1) \} \cap \{ w | \text{Val}(t, S, w, \sigma_2) \} \cap \{ w | \text{Val}(t, S, w, \sigma_3) \} \\
\cap \{ w | \text{Val}(t, S, w, \sigma_4) \}\}
\end{align*}
\]
This is the set of possible worlds in which $S$ is true for each of four assignments, in other words, the set of possible worlds in which both Hanako and Taro taught both John and Mary. The cell determined by the third assignment above is 

$$\{w \mid \text{Val}(t, S, w, \sigma_1)\} \cap \{w \mid \text{Val}(t, S, w, \sigma_2)\} \cap \{w \mid \text{Val}(t, S, w, \sigma_3)\} \cap \{w \mid \text{Val}(f, S, w, \sigma_4)\}.$$  

This is the set of possible worlds in which Hanako taught both John and Mary but Taro taught only Mary.  

In general, each bipartition $\langle X, \Sigma_S - X \rangle$ defined for $S$ determines the set of possible worlds in which the condition expressed by $S$ obtains for those objects assigned by the assignments in $X$ but not for those objects assigned by the rest.  

The partition $\Pi^S$ determined by an interrogative sentence “$S + ka$” is given by  

$$\Pi^S = \{c(X) \mid X \subseteq \Sigma^S\}.$$  

Usually we do not need to consider all the possible worlds but only a certain subset of $W$. As this subset is determined by $S$, let us refer to it by “$W(S)$”. In the present example, $W(S)$ consists of the possible worlds in which teachers are Hanako and Taro and children are John and Mary. Certainly there are other possible worlds such as the ones in which Hanako and Taro are not teachers, or there are other teachers besides them. But they are not relevant to the semantic value of an interrogative sentence “$S + ka$” in the given context.  

It should not be difficult to prove the following proposition, provided that, for any world $w \in W(S)$ and an assignment $\sigma^S$, $S$ is either true or false but not both. Thus, the partition $\Pi^S$ has the properties of exclusiveness and exhaustiveness as before.  

Proposition  

Suppose that $\Pi^S$ is the partition determined by an interrogative sentence “$S + ka$”. Then,  

(i) For any $c_1, c_2 \in \Pi^S$, if $c_1 \neq c_2$ then $c_1 \cap c_2 = \emptyset$, and  

(ii) $\bigcup \Pi^S = W(S)$.  

If $\varnothing \in W(S)$, then $\varnothing$ is in some cell of the partition $\Pi^S$ by (ii). By (i), $\varnothing$ cannot be found in different cells of $\Pi^S$. Thus, if the actual world $\varnothing$ is in $W(S)$,

\[ \forall w \forall \sigma^S \left[ w \in W(S) \rightarrow [\text{Val}(t, S, w, \sigma^S) \lor \text{Val}(f, S, w, \sigma^S)] \right], \]  

and \[ \forall w \forall \sigma^S \left[ w \in W(S) \rightarrow [\neg \text{Val}(t, S, w, \sigma^S) \land \text{Val}(f, S, w, \sigma^S)] \right]. \]
there exists one and only one cell of $\Pi^S$ that contains $\emptyset$. This cell corresponds to
the reality, and hence, it is the true complete answer to the question expressed by “$S + ka$”. Let us record this as a definition.

**Definition: the true complete answer to an interrogative sentence**

(Second version)

If $\Pi^S$ is the partition determined by an interrogative sentence “$S + ka$”, then the true complete answer to “$S + ka$” is a cell $c$ of $\Pi^S$
such that $\emptyset \in c$.

We can define a dc-partition (a partition with a designated cell) for an inter-
rogative sentence “$S + ka$” just as we did in §7.3. It is an ordered pair consisting
of the partition $\Pi^S$ and one of its cells.

There is no need to modify the semantic axiom of an interrogative sentence
we gave in §7.4. We might note that the true complete answer $c^*$ to “$S + ka$”
is the cell of the partition $\Pi^S$ which contains the actual world $\emptyset$. If there is
no cell among the cells of $\Pi^S$ which contains $\emptyset$, then “$S + ka$” will not have a
semantic value. But, in that case, $S$ does not have a semantic value, in the first
place.

Let us see what happens when $S$ is a sentence, namely a closed sentence. In
this case, A cell is either

$$\{w \mid \text{Val}(t, S, w)\},$$

or

$$\{w \mid \text{Val}(f, S, w)\}.$$  

Hence, the partition determined by a yes-no interrogative “$S + ka$” is

$$\{\{w \mid \text{Val}(t, S, w)\}, \{w \mid \text{Val}(f, S, w)\}\}.$$  

Let us denote the proposition expressed by $S$ by “$\overline{S}$” in the style of Montague.
If $W(S) = W$, that is, $S$ has no truth value gap in any possible world, then the
above partition can be written as

$$\{\overline{\overline{S}}, \overline{\overline{S}}\}.$$  

The dc-partition for “$S + ka$” is either

$$\langle \{\overline{\overline{S}}, \overline{\overline{S}}\}, \overline{S} \rangle \text{ or } \langle \{\overline{\overline{S}}, \overline{\overline{S}}\}, \overline{S} \rangle.$$  

$^{24}$ “$\overline{S}$” denotes the negation of $S$. As $w \in W(S)$ and $W(S) = W$,
for any possible world $w$,

$$\text{Val}(t, \overline{S}, w) \leftrightarrow \text{Val}(f, S, w).$$
In our account that does not talk of possible worlds and propositions, the
dc-partition for a yes-no interrogative “S + ka” is either
\[ \langle t | f, t \rangle \text{ or } \langle t | f, f \rangle. \]

What is the difference between the two? For one thing, different yes-no in-
terrogative sentences may have different semantic values in the present account,
while all yes-no interrogative sentences have the same semantic value according
to our original account. The proposition \( \hat{S} \) is a set of possible worlds in which
\( S \) is true, and different sentences determine different sets of possible worlds, be-
cause they might differ in their truth values in different possible worlds. Thus,
we can make a finer distinction between sentences in terms of semantic values.

The same thing can be said with interrogative sentences in general. As a
matter of fact, there is a certain conception of semantics, according to which
two accounts are complementary, one specifying the extensions of interrogative
sentences, and the other their intensions. Not only interrogative sentences but
also expressions of any category should be assigned both extension and intension;
an expression’s extension and intension are related in such a way that the latter
is a function from possible worlds to the former, just as a proposition which is
the intension of a sentence is a function from possible worlds to the truth values
which is the extension of a sentence\(^{25}\).

If we subscribe to such a conception of semantics, then we would retain our
original account of semantic values as an account of extension and supplement
it with a new account of intensions, instead of replacing the former with the
latter.

Such a two-tier semantics might be thought similar to a Fregean semantic
account in terms of \textit{Sinn} and \textit{Bedeutung}. Carnap even thought that his own
method of extension and intension was an improved version of Fregean \textit{Sinn}
and \textit{Bedeutung}\(^{26}\).

But there exists another tradition of interpreting Frege’s distinction without
having recourse to possible worlds. Moreover, we may argue that construing a
Fregean thought (\textit{Gedanke}), which is the \textit{Sinn} of a sentence, as a proposition
in the present sense cannot be the right interpretation.

Suppose we identify a proposition with a set of possible worlds or a function
from possible worlds to truth values. Then, if two sentences coincide in their
truth values in all possible worlds, they must express the same proposition.
But it may well happen that they express different Fregean thoughts. This
possibility is all the more important, considering that Frege’s major interests in
the semantics are centered on the language of mathematics. Take any theorem

\(^{25}\) A proposition as a set of possible worlds and a proposition as a (partial) function from
possible worlds to truth values can be interdefined by the following equivalence.
\[ w \in P \iff P(u) = t. \]

\(^{26}\) See \cite{Carnap 1956}, in particular, §§28–30.
of mathematics. It must be true in all possible worlds. This means that in the
conception of a proposition in terms of possible worlds all true mathematical
sentences should express one and the same proposition, namely, the set of all
possible worlds (or the function which always gives the truth value t for any
possible world). Such a consequence would be disastrous for an analysis of the
language of mathematics.

Thus, Fregean \textit{Sinn} must have a much finer structure than Carnapian or
Montagovian intension could have. Unfortunately Frege has left us only some
tantalizing hints about what such a structure should be. Thus, there might
be various nonequivalent ways of implementing these hints in order to build a
systematic account.

In introducing the distinction between \textit{Sinn} and \textit{Bedeutung}, Frege writes
thus:

\begin{quote}
It is natural, now, to think of there being connected with a sign
(name, combination of words, written mark), besides that which the
sign designates, which may be called the meaning (\textit{Bedeutung}) of the
sign, also what I should call the sense (\textit{Sinn}) of the sign, wherein
the mode of presentation (\textit{die Art des Gegebenseins}) is contained.\textsuperscript{27}
\end{quote}

I wish to follow Frege in associating with an expression not only a semantic
value as its \textit{Bedeutung} but also a way how the semantic value is presented as
its \textit{Sinn}.

We are going to consider indeterminate phrases in indirect discourse, in-
cluding indirect interrogative, in Chapter 10. Although the concept of possible
worlds and semantic theories based on them have been extremely useful in ac-
counting various modal locution, indirect discourse remains a stubborn obstacle
to a successful treatment by them. I believe that Fregean \textit{Sinn} has a better
chance in this area. Thus, what I will associate with an interrogative sentence
as its \textit{Sinn} will not be the same as those intensional objects that are found in
possible world based accounts.

As a semantic account of interrogative sentences which does not explicitly
contain modal vocabulary or construction, there is no substantial difference
between our minimum account of a partition and the one based on possible
worlds except the fact that the latter gives us an intuitively persuasive picture
how an interrogative sentence presents a space of possibilities. The talk of
possible worlds, however, is above all a picture. Can we take it literally and
believe in the real existence of possible worlds other than our world? Of course,
there are some philosophers like David Lewis who do so, but most philosophers
would try to reinterpret the possible world talk in some way.

In this chapter, I have tried to show that a semantics of some simple in-
terrogative construction can be given without committing to the existence of

\textsuperscript{27} \cite{Frege 1984}, p.158.
possible worlds. I hope such an attempt might have some value in a more general project of charting what existential commitments are involved in various parts of our language.
Chapter 8

Entailment between interrogative sentences
Chapter 9

Presupposition of an interrogative sentence
Chapter 10

Indeterminate phrases in indirect discourse
Chapter 11

Indeterminate phrases and demonstratives
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